Online Appendix for

The More the Poorer? Resource Sharing and Scale Economies in Large Families

Rossella Calvi, Jacob Penglase, Denni Tommasi, and Alexander Wolf

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A Proofs

A.1 Proof of Lemma 1

From the definition of indifference scales in Equation (7),

$$V_j(\alpha_t + p, x + \ln \eta_{j,t}) = V_j(\alpha_{t'} + p, x + \ln \eta_{j,t'} - \ln I_{j,t',t}).$$

It follows from Equation (4) that

$$V_j(\alpha_t + p, x + \ln \eta_{j,t}) = V_j(\alpha_{t'} + p, x + \ln \eta_{j,t} - \ln s_{j,t} + \ln s_{j,t'}).$$

Thus,

$$V_j(\alpha_{t'} + p, x + \ln \eta_{j,t'} - \ln I_{j,t',t}) = V_j(\alpha_{t'} + p, x + \ln \eta_{j,t} - \ln s_{j,t} + \ln s_{j,t'}). \tag{A1}$$

For Equation (A1) to hold, the following needs to be true

$$\ln I_{j,t',t} = \ln \eta_{j,t'} - \ln \eta_{j,t} + \ln s_{j,t} - \ln s_{j,t'}.$$

A.2 Proof of Theorem 1

We rewrite the Engel curves in terms of expenditure (instead of log expenditure). Let y be household expenditure with $x = \ln y$. Define $\Omega^{k_j}_{j,n}(y) = W^{k_j}_{j,n}(x)$, $\omega^{k_j}_{j,n}(y\eta_{j,n}/s_{j,n}) = w^{k_j}_{j,n}(x + \ln \eta_{j,n} - \ln s_{j,n})$. Identification will come from the derivates of the Engel curves, so let $\nu^{k_j}_{j,n}(y) = \nabla_y \omega^{k_j}_{j,n}(y)$ and $\zeta^{k_j}_{j,n}(y) = \nabla^2_y \omega^{k_j}_{j,n}(y)$.

Consider a one-child household, i.e., a nuclear household with one adult man m, one adult woman w, and one child c ($n = 1$). From Equation (5),

$$w^{k_j}_{j,1}(\alpha_1 + p, x + \ln \eta_{j,1}(\alpha_1, p)) = \lambda^{k_j}_{j,1}(\alpha_1, p) + w^{k_j}_{j,1}(p, x + \ln \eta_{j,1}(\alpha_1, p) - \ln s_{j,1}(\alpha_1, p)).$$

for $j = m, w, c$. We use one-child households as the reference household for all person types. Under a single price regime, Assumption 5, and appropriate normalizations, the household-level budget shares for private assignable goods k_j for one-child households are given by

$$\Omega^{k_j}_{j,1}(y) = \eta_{j,1} \omega^{k_j}_{j,1}(\eta_{j,1} y). \tag{A2}$$

Recall that $\Omega^{k_j}_{j,1}$ is observed. Given Assumptions 3.i and 3.ii, resource shares and the second derivatives of the individual budget shares are identified by the following set of four equations in four unknowns:

$$\nabla^2_y \Omega^{k_j}_{j,1}(y)\big|_0 = \eta_{j,1} \zeta^{k_j}(0),$$

for $j = m, w, c$ with $\eta_{c,1} = 1 - \eta_{m,1} - \eta_{w,1}$. Then, taking the first derivative of Equation (A2) with respect to y and applying Assumption 3.i yields:

$$\nabla_y \Omega^{k_j}_{j,1}(y)\big|_0 = \eta_{j,1} \nu^{k_j}_j(0),$$

which identifies $\nu^{k_j}_j(0)$, for $j = m, w, c$.

Moving onto n-child couples with $n > 1$ (nuclear households with one man (m), one woman (w), and $n > 1$ children), the household-level budget shares for private assignable goods k_j are given by:

$$\Omega_{j,n}^{k_j}(y) = \eta_{j,n}\left[\lambda_{j,n}^{k_j} + \omega_{j,n}^{k_j}\left(\frac{\eta_{j,n}y}{s_{j,n}}\right)\right], \tag{A3}$$

for $j = m, w$, and

$$\Omega_{c,n}^{k_c}(y) = n\eta_{c,n}\left[\lambda_{c,n}^{k_c} + \omega_{c,n}^{k_j}\left(\frac{\eta_{c,n}y}{s_{c,n}}\right)\right], \tag{A4}$$

for children. Differentiating Equations (A3) and (A4) with respect to y:

$$\nabla_y \Omega_{j,n}^{k_j}(y)\big|_0 = \frac{\eta_{j,n}^2}{s_{j,n}} \nu_j^{k_j}(0) \tag{A5}$$

for $j = m, w$, and for children

$$\nabla_y \Omega_{c,n}^{k_c}(y)\big|_0 = n\frac{\eta_{c,n}^2}{s_{c,n}} \nu_c^{k_c}(0). \tag{A6}$$

Differentiating again yields:

$$\nabla_y^2 \Omega_{j,n}^{k_j}(y)\big|_0 = \frac{\eta_{j,n}^3}{s_{j,n}^2} \zeta^{k_j}(0) \tag{A7}$$

for $j = m, w$, and for children

$$\nabla_y^2 \Omega_{c,n}^{k_c}(y)\big|_0 = n\frac{\eta_{c,n}^3}{s_{c,n}^2} \zeta^{k_c}(0). \tag{A8}$$

Since $\nu_j^{k_j}(0)$ and $\zeta_j^{k_j}(0)$ are known, the ratio of the second and first derivatives identifies $\kappa_{j,n} = \frac{\eta_{j,n}}{s_{j,n}}$:

$$\frac{\nabla_y^2 \Omega_{j,n}^{k_j}(y)\big|_0}{\nabla_y \Omega_{j,n}^{k_j}(y)\big|_0} = \frac{\eta_{j,n}\zeta^{k_c}(0)}{s_{j,n}\nu_j^{k_c}(0)} = \kappa_{j,n}\frac{\zeta^{k_c}(0)}{\nu_j^{k_c}(0)}.$$

Substituting $\kappa_{j,n}$ into equations Equations (A5) and (A6) yields:

$$\nabla_y \Omega_{j,n}^{k_j}(y)\big|_0 = \eta_{j,n}\kappa_{j,n}\nu_j^{k_j}(0).$$

for $j = m, w$, and for children

$$\nabla_y \Omega_{c,n}^{k_c}(y)\big|_0 = n\eta_{c,n}\kappa_{c,n}\nu_c^{k_c}(0),$$

which identify $\eta_{j,n}$ and, in turn, $s_{j,n}$ from $\kappa_{j,n}$.

A.3 Proof of Theorem 2

Let $\tau = 1$ denote the household where one of all J person types are present. This will be our reference household. Scale economies will then be measured relative to this household. We start by proving identification in the reference household. Next, we prove identification in a household with a different composition and generalize this to all other household types. So, the structure of the proof follows closely the proof of Theorem 1.

We rewrite the Engel curves in terms of expenditure as follows. Let y be household expenditure with $x = \ln y$. Define $\Omega^{k_j}_{j,\tau}(y) = W^{k_j}_{j,\tau}(x)$, $\omega^{k_j}_{j,\tau}(y\eta_{j,\tau}/s_{j,\tau}) = w^{k_j}_{j,\tau}(x + \ln\eta_{j,\tau} - \ln s_{j,\tau})$. Identification will come from the derivates of the Engel curves, so let $\nu^{k_j}_{j,\tau}(y) = \nabla_y \omega^{k_j}_{j,\tau}(y)$ and $\zeta^{k_j}_{j,\tau}(y) = \nabla^2_y \omega^{k_j}_{j,\tau}(y)$.

Let $\tau = 1$ denote the reference household, where all J person types are present. From Equation (5),

$$w^{k_j}_{j,1}(\alpha_1 + p, x + \ln\eta_{j,1}(\alpha_1, p)) = \lambda^{k_j}_{j,1}(\alpha_1, p) + w^{k_j}_{j,1}(p, x + \ln\eta_{j,1}(\alpha_1, p) - \ln s_{j,1}(\alpha_1, p)).$$

Under a single price regime, Assumption 5, and appropriate normalizations, the household-level budget share functions for private assignable goods k_j for person type j in the reference household is given by:

$$\Omega^{k_j}_{j,1}(y) = \eta_{j,1}\omega^{k_j}_{j,1}(\eta_{j,1}y). \tag{A9}$$

Given Assumptions 3.i and 3.ii, resource shares and the second derivatives of the individual budget share functions are identified by the following set of $J+1$ equations in $J+1$ unknowns:[1]

$$\nabla^2_y \Omega^{k_j}_{j,1}(y)\big|_0 = \eta_{j,1}\zeta^{k_j}(0),$$

with $\sum_j n_j\eta_{j,1} = 1$. Taking the first derivative of Equation (A9) with respect to y and applying Assumption 3.i yields:

$$\nabla_y \Omega^{k_j}_{j,1}(y)\big|_0 = \eta_{j,1}\nu^{k_j}_j(0),$$

which identifies $\nu^{k_j}_j(0)$ for all j.

Turning to the non-reference households of types $\tau = t \neq 1$, the household-level budget shares for the private assignable goods k_j are given by:

$$\Omega^{k_j}_{j,t}(y) = n_j\eta_{j,t}\left[\lambda^{k_j}_{j,t} + \omega^{k_j}_{j,t}\left(\frac{\eta_{j,t}y}{s_{j,t}}\right)\right]. \tag{A10}$$

Differentiating Equation (A10) with respect to y and imposing Assumption 3.i:

$$\nabla_y \Omega^{k_j}_{j,t}(y)\big|_0 = n_j\frac{\eta^2_{j,t}}{s_{j,t}}\nu^{k_j}_j(0). \tag{A11}$$

Note that $\nu^{k_j}_j(0)$ does not depend on t. Differentiating again yields:

$$\nabla^2_y \Omega^{k_j}_{j,t}(y)\big|_0 = n_j\frac{\eta^3_{j,t}}{s^2_{j,t}}\zeta^{k_j}(0).$$

The ratio of the second and first derivatives above identifies $\kappa_{j,t} = \frac{\eta_{j,t}}{s_{j,t}}$:

$$\frac{\nabla^2_y \Omega^{k_j}_{j,t}(y)\big|_0}{\nabla_y \Omega^{k_j}_{j,t}(y)\big|_0} = \frac{\eta_{j,t}\zeta^{k_j}(0)}{s_{j,t}\nu^{k_j}_j(0)}.$$

Substituting $\kappa_{j,t}$ into Equation (A11) for all $j = 1, \ldots J$ identifies $\eta_{j,t}$ and $s_{j,t}$.

[1] Assumption 3.i is generalized such that the similarity across sizes is a similarity across all τ household types. Similarly, Assumption 3.ii is generalized such that the similarity in preferences across men, women, and children is now a similarity in preferences across all J person types.

A.4 Proof of Corollary 1

Let $w_{j,n}^{k_j}(\alpha_n + p, x + \ln(\eta_{j,n}))$ be a polynomial of degree l in x. Then there are a finite number (i.e., l) of non-zero derivatives. Let $\zeta_j^{k_j} = \nabla_x^l w_{j,n}^{k_j}(x)$, where $\zeta_j^{k_j}$ does not depend on x or household type n. Furthermore, let $v_j^{k_j} = \nabla_x^{l-1} w_{j,n}^{k_j}(x) = \gamma_j^{k_j} + \zeta_j^{k_j} x$.

We take the l^{th} derivative of $W_{j,n}^{k_j}(x)$ with respect to x:

$$\nabla_x^l W_{j,n}^{k_j}(x) = \eta_{j,n} \zeta_j^{k_j} \tag{A12}$$

With three household sizes (e.g., $n = 1, 2, 3$), Equation (A12) results in a system of nine Engel curves with nine unknowns: three preference parameters $\zeta_j^{k_j}$ and six resource shares (since resource shares sum to one, we only need to identify two resource shares for households with children). We can then use Equation (A12) to identify $\zeta_j^{k_j}$ and $\eta_{j,n}$.

Given $\zeta_j^{k_j}$ and $\eta_{j,n}$, and under the appropriate normalizations to scale economies in the reference households, the $(l-1)^{th}$ derivatives of $W_{j,n}^{k_j}(x)$ in one-child families identifies the preference parameters $\gamma_j^{k_j}$ for $j = m, w, c$:

$$\nabla_x^{l-1} W_{j,1}^{k_j}(x) = \eta_{j,1} \left[\gamma_j^{k_j} + \zeta_j^{k_j} (x + \ln \eta_{j,1}) \right], \tag{A13}$$

Preference parameters $\gamma_j^{k_j}$ and $\zeta_j^{k_j}$ are both assumed to not vary across household type, and are therefore treated as known in the non-reference households. Moreover, resource shares are identified for all household types from Equation (A12). What is left to identify are scale economies in the non-reference households.

To identify $s_{j,n}$, we take the $(l-1)^{th}$ derivative of $W_{j,n}^{k_j}(x)$ with respect to x:

$$\nabla_x^{l-1} W_{j,n}^{k_j}(x) = \eta_{j,n} \left[\gamma_j^{k_j} + \zeta_j^{k_j} (x - \ln s_{j,n} + \ln \eta_{j,n}) \right] \tag{A14}$$

For each person type j in a household of size $n > 1$, Equation (A14) is one equation with one unknown. We can therefore solve for $s_{j,n}$.

B Additional Tables and Figures

Table A1: Pre-Estimation Tests of Assignable Goods

	Slope				Curvature			
	Estimate	|t-stat|	Mean	Median	Estimate	|t-stat|	Mean	Median
Bangladesh:								
Men's Food (Cereals, Pulse, Vegetables)	-0.159	5.032	3.454	3.482	0.004	3.281	2.993	3.025
Women's Food (Cereals, Pulse, Vegetables)	-0.086	2.810	3.371	3.351	0.002	1.214	2.958	2.928
Children's Food (Cereals, Pulse, Vegetables)	-0.123	4.082	2.434	2.262	0.004	2.779	2.100	1.913
Men's Food (Meat, Fish, Milk Products)	0.076	2.016	1.781	1.519	-0.002	1.130	1.675	1.435
Women's Food (Meat, Fish, Milk Products)	0.010	0.306	1.536	1.247	0.001	0.605	1.416	1.123
Children's Food (Meat, Fish, Milk Products)	0.077	2.054	1.725	1.594	-0.002	1.166	1.607	1.474
Men's Food (All)	0.019	0.306	0.946	0.857	-0.002	0.624	0.966	0.892
Women's Food (All)	0.041	0.734	1.123	0.943	-0.002	0.931	1.126	0.949
Children's Food (All)	0.012	0.206	1.557	1.380	-0.001	0.208	1.551	1.379
Men's Clothing	0.009	1.235	0.970	0.853	-0.001	1.704	0.979	0.847
Women's Clothing	-0.041	4.995	1.430	1.266	0.002	4.503	1.340	1.189
Children's Clothing	-0.007	0.797	0.955	0.835	0.000	0.560	0.962	0.848
Mexico:								
Men's Clothing	-0.012	4.376	1.116	0.967	0.001	5.197	1.167	1.027
Women's Clothing	-0.015	5.668	1.675	1.489	0.001	6.809	1.758	1.525
Children's Clothing	-0.018	4.336	1.592	1.300	0.001	4.743	1.602	1.308

Note: OLS estimates. Slope coefficients and curvature parameters of the fully-interacted linear regression model (12). We also report the t-statistics of the slope coefficients and curvature parameters as well as the mean and median of the empirical distributions. The predicted values are obtained from a regression of the assignable good budget shares on preference factors $(X_h, X_h^\tau)'$, $(X_h, X_h^\tau)' x_h$, and $(X_h, X_h^\tau)' x_h^2$. We report the predicted curvature of the assignable good Engel curves evaluated at the mean values of $(X_h, X_h^\tau)'$.

Table A2: Resource Shares and Scale Economies: Bangladesh

	Resource Shares				Scale Economies	
	Children		Women			
	Estimate	SE	Estimate	SE	Estimate	SE
	(1)	(2)	(3)	(4)	(5)	(6)
Number of Men	-0.252***	(0.0430)	-0.267***	(0.0397)	0.626*	(0.354)
Number of Women	-0.328***	(0.0405)	0.416***	(0.0401)	1.415***	(0.473)
Number of Children	0.186***	(0.0276)	-0.0783***	(0.0226)	-1.211***	(0.167)
Average Age Men	0.00110	(0.00268)	0.000604	(0.00205)	-0.00689	(0.0137)
Average Age Women	0.00126	(0.00364)	0.000902	(0.00272)	0.0857***	(0.0316)
Average Age Children	0.0731***	(0.0101)	-0.0473***	(0.00803)	-0.558***	(0.129)
Head Woman Works	-0.0166	(0.0692)	0.0301	(0.0477)	-0.204	(0.343)
Head Man Works	-0.0104	(0.112)	-0.0389	(0.0877)	1.489***	(0.405)
Average Education Women	-0.000910	(0.0299)	-0.0136	(0.0184)	0.101	(0.144)
Average Education Men	-0.0723***	(0.0250)	0.0258	(0.0169)	0.0682	(0.142)
Work in Rural Area	0.0131	(0.0883)	-0.105	(0.0647)	0.134	(0.376)
Dhaka	0.0785	(0.0642)	0.0764*	(0.0443)	0.861**	(0.413)
Year=2011	0.0971	(0.0591)	-0.124***	(0.0464)	-0.0941	(0.329)
Constant	-0.689***	(0.169)	-0.645***	(0.132)	3.808***	(1.096)
Sample Size	6,442					

Note: BIHS data (2011/12 and 2015). NLSUR estimates conditional on a set of observable household characteristics and composition variables. Robust standard errors in parentheses. We specify resource shares and scale economies using an inverse logistic function that guarantees that they are bounded between zero and one. We restrict scale economies to be identical across men, women, and children. Age variables are divided by 10 to ease computation.

Table A3: Resource Shares and Scale Economies: Mexico

	Resource Shares				Scale Economies	
	Children		Women			
	Estimate	SE	Estimate	SE	Estimate	SE
	(1)	(2)	(3)	(4)	(5)	(6)
Number of Men	-0.0173	(0.0636)	-0.2048***	(0.0515)	-0.3317	(0.3179)
Number of Women	0.0687	(0.0531)	0.0776	(0.0512)	-0.0777	(0.2627)
Number of Boys	-0.1126***	(0.0389)	0.0634**	(0.0323)	0.8150**	(0.3622)
Number of Girls	-0.1169***	(0.0399)	0.0122	(0.0335)	0.8554**	(0.3982)
Average Age Men	0.1192***	(0.0374)	0.0928***	(0.0324)	-0.4914***	(0.1804)
Average Age Women	-0.0885*	(0.0486)	-0.1386***	(0.0387)	-0.2543	(0.2361)
Average Age Children	0.2774***	(0.1024)	-0.1145	(0.0839)	-0.3430	(0.4089)
Head Woman Works	-0.0610	(0.0900)	0.1278*	(0.0667)	-0.0789	(0.4902)
Head Man Works	-0.0212	(0.1566)	-0.1236	(0.1202)	0.5091	(0.4989)
Average Education Women	0.0816	(0.1053)	0.0249	(0.0838)	-0.8639	(0.5852)
Average Education Men	0.2180**	(0.0967)	-0.0566	(0.0814)	0.1155	(0.4369)
Urban	-0.0443	(0.0832)	-0.0149	(0.0701)	-0.5006	(0.4430)
Mexico City	0.2725	(0.1817)	-0.4227**	(0.1720)	-1.0457*	(0.5936)
Constant	-1.1660***	(0.2659)	-0.1974	(0.1869)	2.0648	(1.4016)
Sample Size	36,075					

Note: ENIGH data (2018). NLSUR estimates conditional on a set of observable household characteristics and composition variables. Robust standard errors in parentheses. We specify resource shares and scale economies using an inverse logistic function that guarantees that they are bounded between zero and one. We restrict scale economies to be identical across men, women, and children. Age variables are divided by 10 to ease computation.

Table A4: Estimated Resource Shares and Scale Economies: Representative Household

	Bangladesh		Mexico	
	Estimate	Std. Error	Estimate	Std. Error
	(1)	(2)	(3)	(4)
A) Resource Shares: $\widehat{\eta}_{j,\tau}$				
Children	0.298	0.016	0.249	0.024
Women	0.286	0.009	0.415	0.021
Men	0.417	0.016	0.336	0.018
B) Scale Economies: $\widehat{s}_{\tau}$	0.992	0.005	0.959	0.031

Note: The table reports the estimated resource shares and scale economies in a representative non-reference household, defined as non-reference household with all preference factors and household type variables at their median value (see Tables 2 and 3 for a list of these values).

Table A5: Estimated Scale Economies by Person Type (Bangladesh)

	Obs.	Mean	Median	Std. Dev.
	(1)	(2)	(3)	(4)
A) Resource Shares: $\widehat{\eta}_{j,\tau}$				
Children	6,442	0.128	0.073	0.069
Women	6,442	0.263	0.277	0.075
Men	6,442	0.338	0.368	0.089
B) Scale Economies: $\widehat{s}_{j,\tau}$				
Children	5,370	0.943	0.995	0.124
Women	5,370	0.957	0.974	0.053
Men	5,370	0.997	1.000	0.032

Note: The table reports the scale economies estimated separately for children, women, and men.

Table A6: Robustness Checks: Endogeneity and Selection

	Mean	St. Dev.	Min.	Median	Max.
	(1)	(2)	(3)	(4)	(5)
A) Bangladesh					
Endogeneity: Household Type					
Children's Resource Share	0.120	0.053	0.014	0.110	0.332
Women's Resource Share	0.312	0.092	0.105	0.355	0.488
Men's Resource Share	0.325	0.092	0.108	0.347	0.550
Scale Economies	0.956	0.075	0.154	0.984	1.000
Endogeneity: Household Expenditure	0.140	0.091	0.010	0.114	0.692
Children's Resource Share	0.265	0.089	0.071	0.282	0.459
Women's Resource Share	0.351	0.122	-0.001	0.370	0.618
Men's Resource Share	0.965	0.063	0.170	0.989	1.000
Scale Economies					
B) Mexico					
Endogeneity: Household Type					
Children's Resource Share	0.163	0.091	0.013	0.129	0.490
Women's Resource Share	0.321	0.111	0.050	0.367	0.619
Men's Resource Share	0.297	0.088	0.000	0.320	0.623
Scale Economies	0.916	0.111	0.163	0.955	1.000
Endogeneity: Household Expenditure					
Children's Resource Share	0.150	0.091	0.007	0.119	0.513
Women's Resource Share	0.333	0.127	0.035	0.375	0.709
Men's Resource Share	0.294	0.083	0.000	0.297	0.648
Scale Economies	0.971	0.076	0.024	0.994	1.000

Note: This table reports the estimated resource shares and scale economies from specifications that account for endogeneity in household type (e.g., selection) and endogeneity in household expenditure (e.g., measurement error).

Table A7: Poverty Rates (%) By Household Size in Bangladesh (No Age Adjustment)

	Household Size					
	All	3	4	5	6	7+
A) Individual Expenditure ($\widehat{\eta}_{j,\tau}y$)						
Children	23.93	1.67	10.30	22.98	34.64	44.64
	[16.98-30.87]	[0.17-3.16]	[5.39-15.21]	[14.74-31.22]	[25.34-43.93]	[34.63-54.64]
Women	0.81	0.00	0.26	0.64	1.41	1.84
	[0.39-1.24]	[0.00-0.04]	[0.12-0.39]	[0.27-1.00]	[0.76-2.06]	[0.73-2.95]
Men	0.94	0.37	0.54	0.94	1.26	1.67
	[0.50-1.39]	[0.00-0.80]	[0.18-0.89]	[0.51-1.37]	[0.69-1.83]	[0.90-2.43]
B) Individual Consumption ($\widehat{\eta}_{j,\tau}y/\widehat{s}_{\tau}$)						
Children	23.01	1.67	9.18	22.02	33.43	43.92
	[16-30.03]	[0.17-3.16]	[4.37-13.98]	[13.75-30.29]	[23.89-42.96]	[33.53-54.31]
Women	0.61	0.00	0.14	0.36	1.08	1.59
	[0.24-0.97]	[0-0.04]	[0.03-0.24]	[0.09-0.63]	[0.44-1.72]	[0.63-2.55]
Men	0.65	0.37	0.34	0.56	0.89	1.21
	[0.22-1.07]	[0-0.8]	[0.06-0.62]	[0.14-0.98]	[0.32-1.47]	[0.37-2.04]

Notes: The table presents poverty rates by household size for Bangladesh computed using two approaches: model-based individual expenditure and model-based individual consumption. Individual expenditure is obtained by multiplying total annual household expenditure by individual resource shares. Individual consumption is obtained by dividing individual expenditure by scale economies. Poverty lines correspond to the 1.90 dollar/day poverty line for men, women, and children.

Table A8: Poverty Rates (%) By Household Size in Mexico (No Age Adjustment)

	Household Size					
	All	3	4	5	6	7+
A) Individual Expenditure ($\widehat{\eta}_{j,\tau}y$)						
Children	23.93	1.67	10.30	22.98	34.64	44.64
	[16.98-30.87]	[0.17-3.16]	[5.39-15.21]	[14.74-31.22]	[25.34-43.93]	[34.63-54.64]
Women	0.81	0.00	0.26	0.64	1.41	1.84
	[0.39-1.24]	[0-0.04]	[0.12-0.39]	[0.27-1]	[0.76-2.06]	[0.73-2.95]
Men	0.94	0.37	0.54	0.94	1.26	1.67
	[0.5-1.39]	[0-0.8]	[0.18-0.89]	[0.51-1.37]	[0.69-1.83]	[0.9-2.43]
B) Individual Consumption ($\widehat{\eta}_{j,\tau}y/\widehat{s}_{\tau}$)						
Children	23.93	1.67	10.30	22.98	34.64	44.64
	[16.00-30.03]	[0.17-3.16]	[4.37-13.98]	[13.75-30.29]	[23.89-42.96]	[33.53-54.31]
Women	0.81	0.00	0.26	0.64	1.41	1.84
	[0.24-0.97]	[0.00-0.04]	[0.03-0.24]	[0.09-0.63]	[0.44-1.72]	[0.63-2.55]
Men	0.94	0.37	0.54	0.94	1.26	1.67
	[0.22-1.07]	[0.00-0.80]	[0.06-0.62]	[0.14-0.98]	[0.32-1.47]	[0.37-2.04]

Notes: The table presents poverty rates by household size for Mexico computed using two approaches: model-based individual expenditure and model-based individual consumption. Individual expenditure is obtained by multiplying total annual household expenditure by individual resource shares. Individual consumption is obtained by dividing individual expenditure by scale economies. Poverty lines correspond to the 1.90 dollar/day poverty line for men, women, and children.

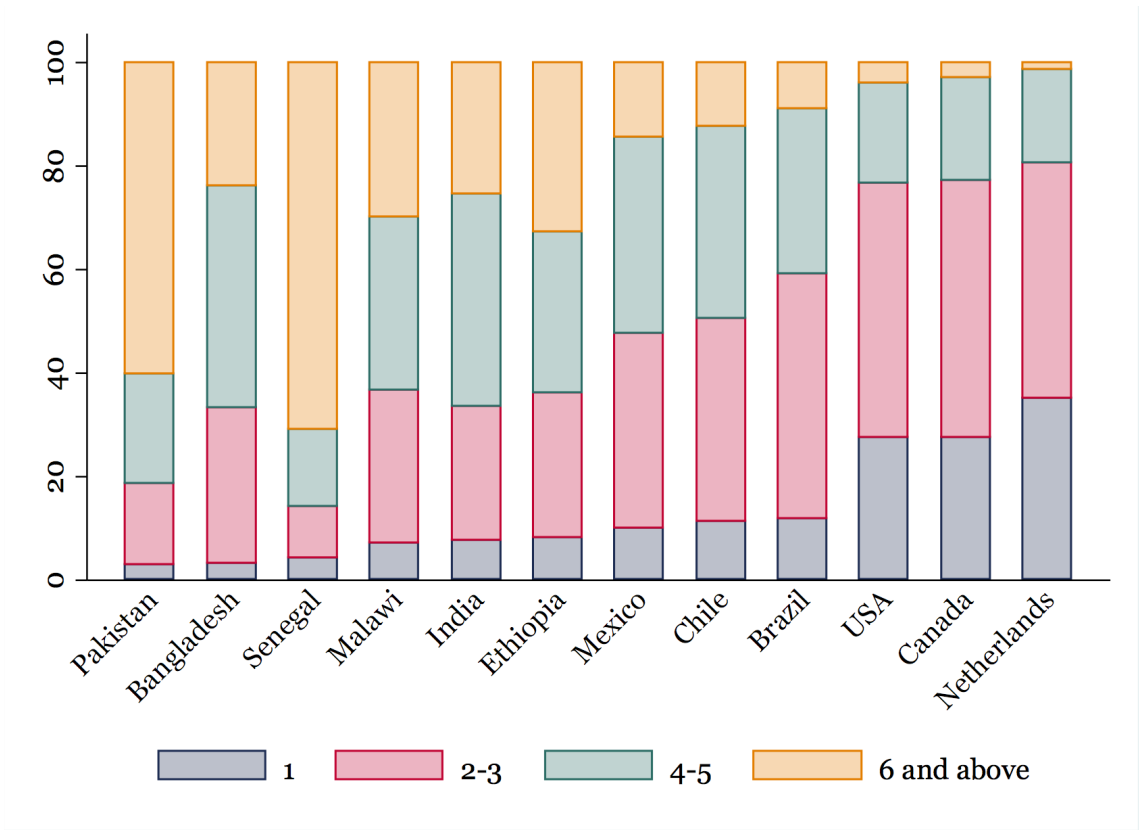
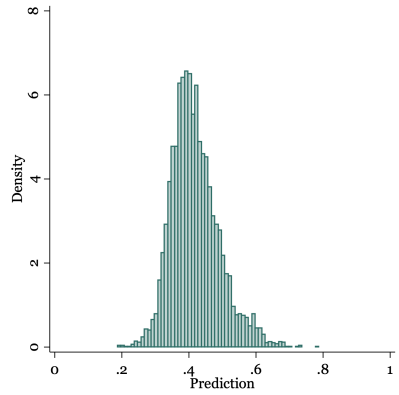


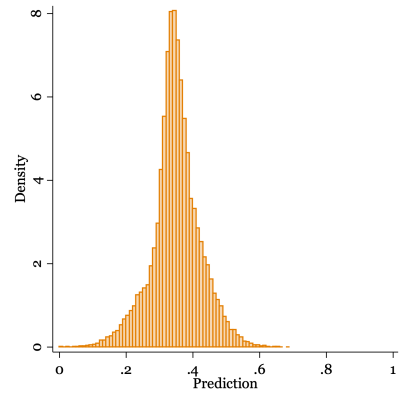
Figure A1: Household Sizes in Selected Countries

Note: Authors' calculations based on the most recent IPUMS data.

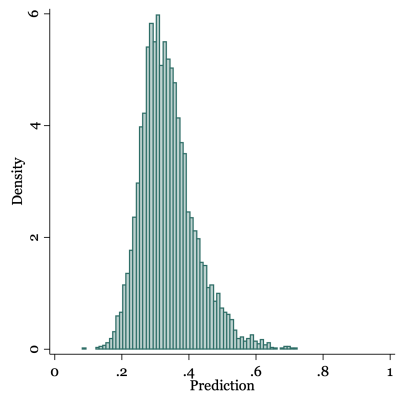
Figure A2: Estimated Resource Shares and Scale Economies: Empirical Distributions



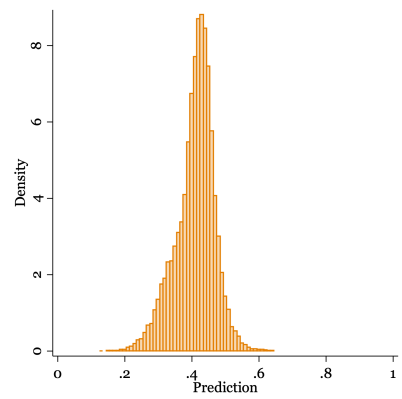
(A.1) Bangladesh: Men's Resource Shares ($M\eta_{m,\tau}$)



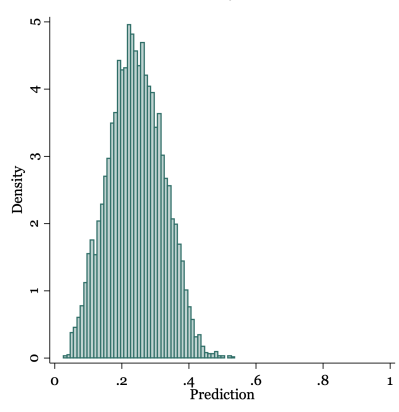
(A.1) Mexico: Men's Resource Shares ($M\eta_{m,\tau}$)



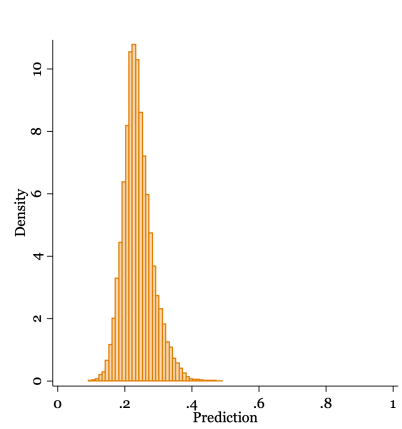
(B.1) Bangladesh: Women's Resource Shares ($W\eta_{w,\tau}$)



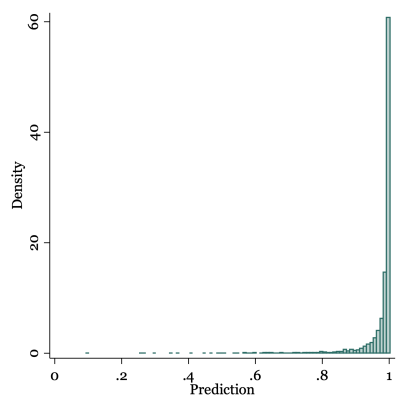
(B.1) Mexico: Women's Resource Shares ($W\eta_{w,\tau}$)



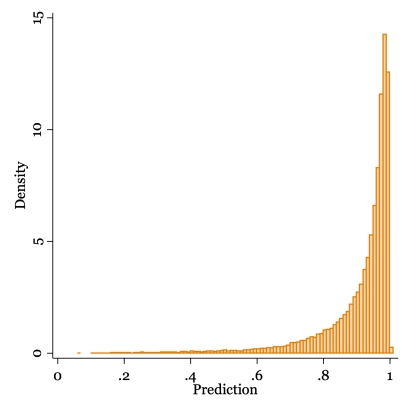
(C.1) Bangladesh: Children's Resource Shares ($C\eta_{c,\tau}$)



(C.1) Mexico: Children's Resource Shares ($C\eta_{c,\tau}$)



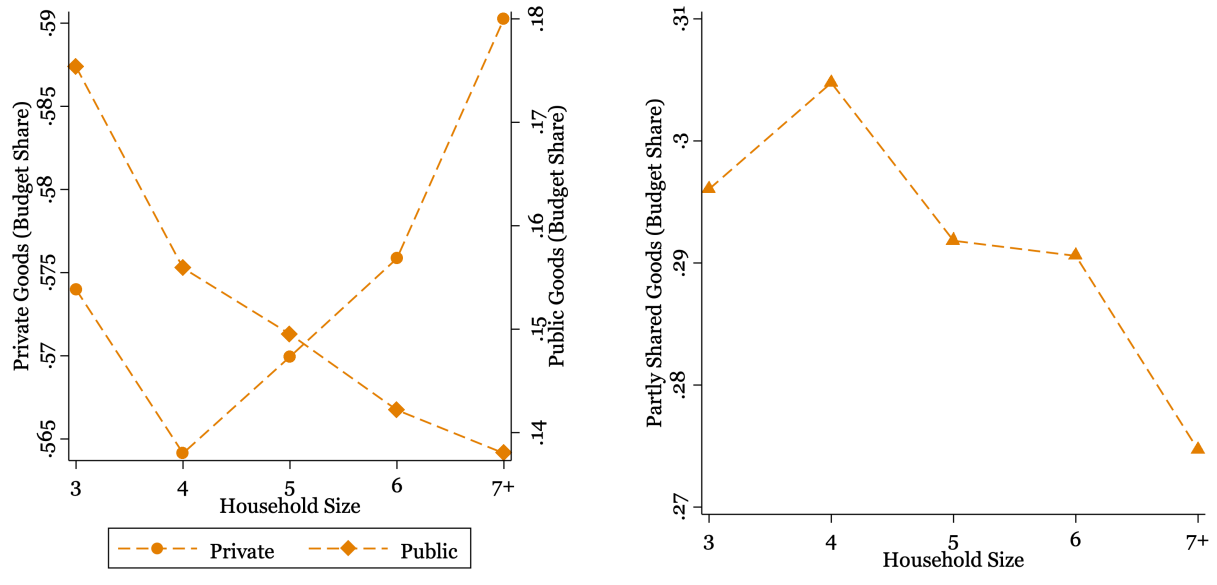
(D.1) Bangladesh: Economies of Scale (s_τ)



(D.2) Bangladesh: Economies of Scale (s_τ)

Note: Empirical distributions of the estimated resource shares and scale economies. The summary statistics are presented in Table 4.

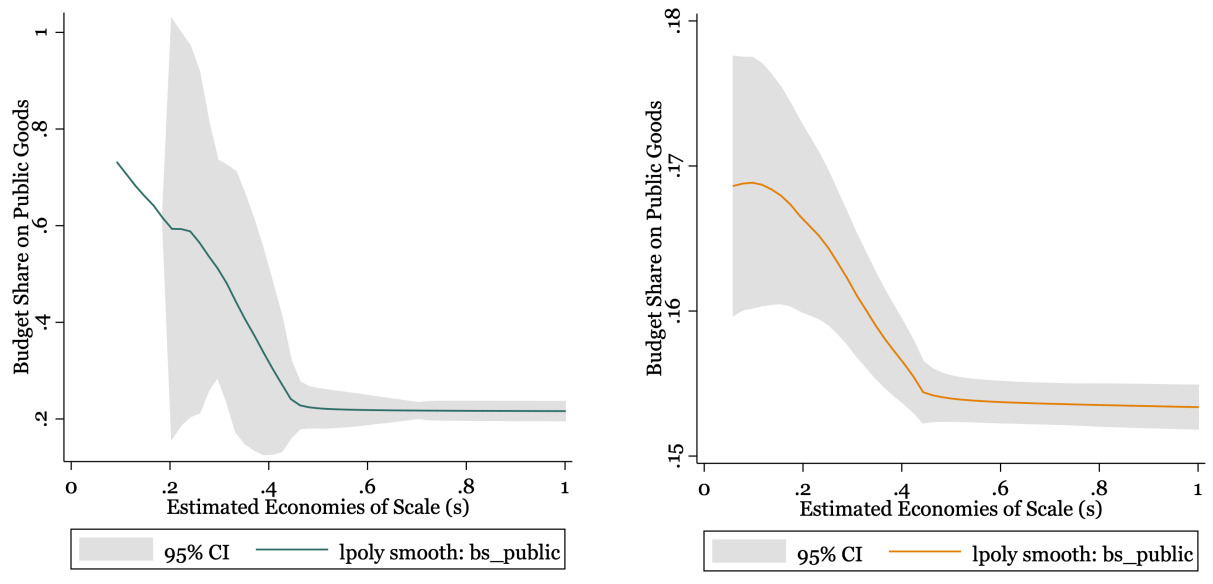
Figure A3: Budget Shares on Private, Public, and Sharable Goods: Mexico



(A) Public and Private Goods (B) Sharable Goods

Note: Share of budget allocated to private and public goods in Mexico. Private goods include food, clothing, footwear, and personal accessories. Public goods include rent, house/residence expenditure, and energy. Sharable goods include fuel for transportation and other expenses on vehicles (excl. purchase).

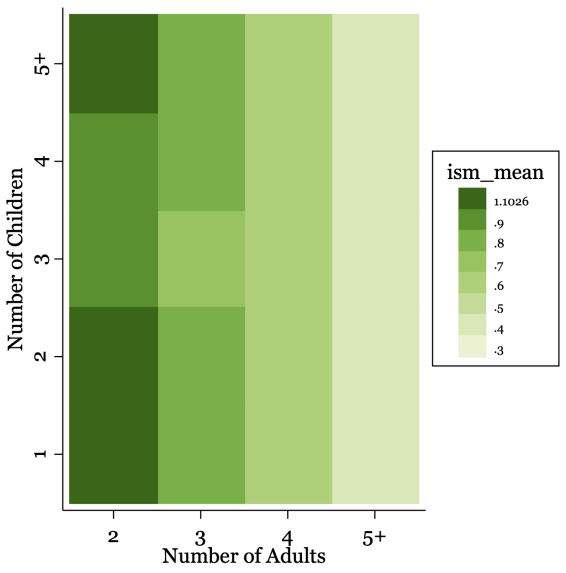
Figure A4: Public Goods and Estimated Economies of Scale



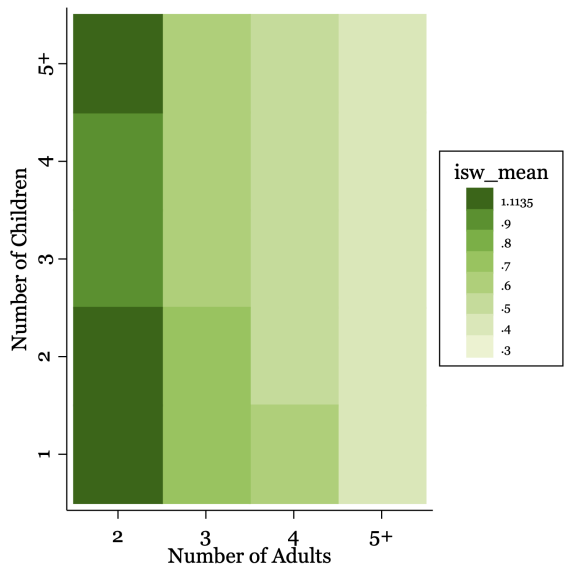
(A) Bangladesh (B) Mexico

Note: Local polynomial regressions to examining the correlation between scale economies and the consumption of public goods. Public goods include rent, house/residence expenditure, and energy. It is reassuring that we estimate larger scale economies (lower $\widehat{s}_\tau$) when the budget share on public goods is large.

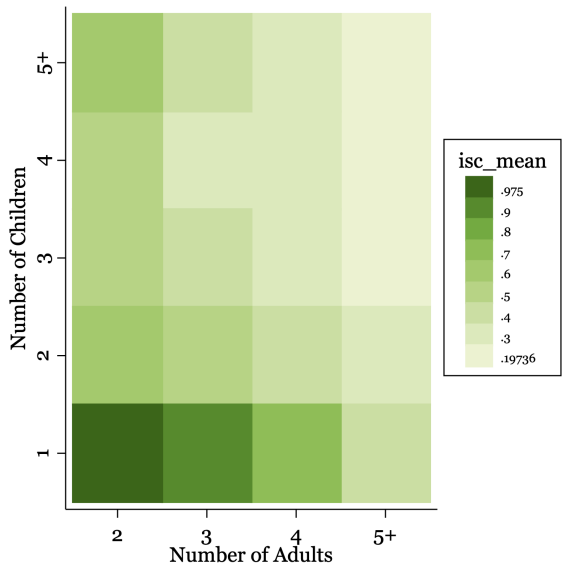
Figure A5: Indifference Scales by Household Composition: Bangladesh



(A) Men



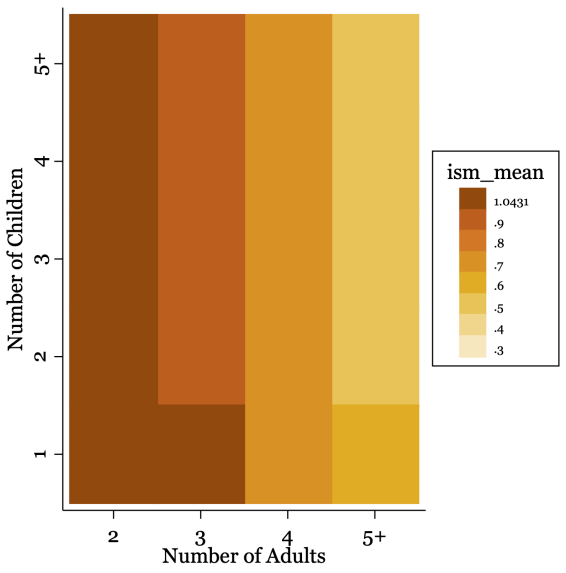
(B) Women



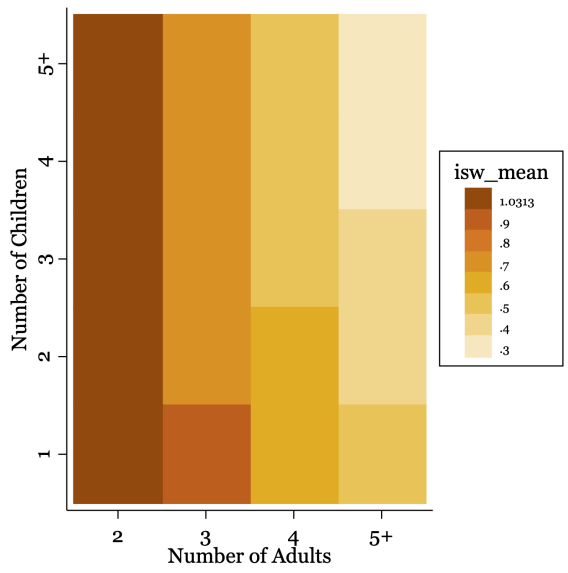
(C) Children

Note: The figure shows how the average indifference scales for children, women and men vary by the number of children and adults in the family. Indifference scales are computed following Lemma 1. The reference household is defined as having one of each person type (here, this implies one man, one woman, one child) Adults' indifference scales decline substantially as the number of adults increases. By contrast, for a given number of adults, the decline is minimal as the number of children increases. Children's indifference scales decline in both the number of children and adults.

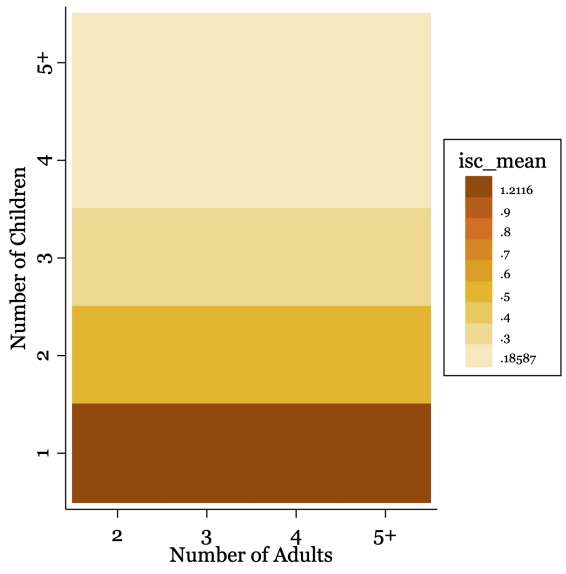
Figure A6: Indifference Scales by Household Composition: Mexico



(A) Men



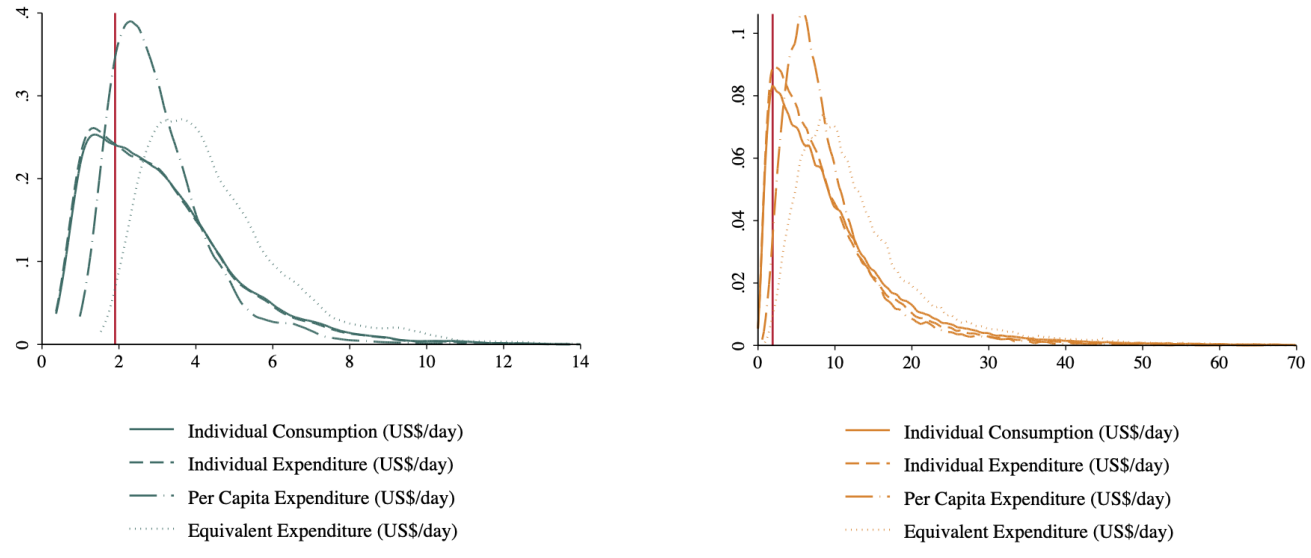
(B) Women



(C) Children

Note: The figure shows how the average indifference scales for children, women and men vary by the number of children and adults in the family. Indifference scales are computed following Lemma 1. The reference household is defined as having one of each person type (here, this implies one man, one woman, one child) Adults' indifference scales decline substantially as the number of adults increases. By contrast, for a given number of adults, the decline is minimal as the number of children increases. Children's indifference scales decline in the number of children.

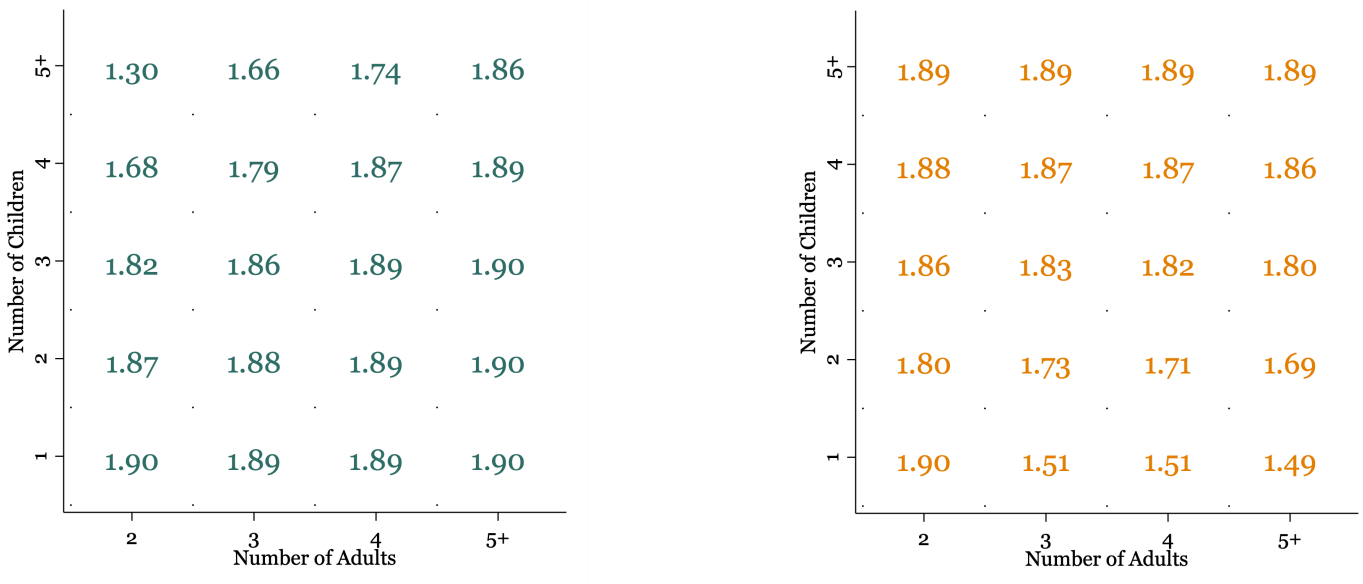
Figure A7: Empirical Distributions



(A) Bangladesh (B) Mexico

Note: The figure presents the empirical distributions of household per-capita expenditure, household expenditure adjusted using the OECD equivalence scale, model-based individual expenditure and model-based individual consumption for Bangladesh and Mexico. Per Capita expenditure is computed by dividing household expenditure by household size. To compute individual expenditure, we multiply household expenditure by each person type's resource share. Individual consumption is equal to equal to expenditure divided by scale economies. In Bangladesh, only households in 2015 are included. The vertical line corresponds to the percentile of the US$1.90/day poverty line.

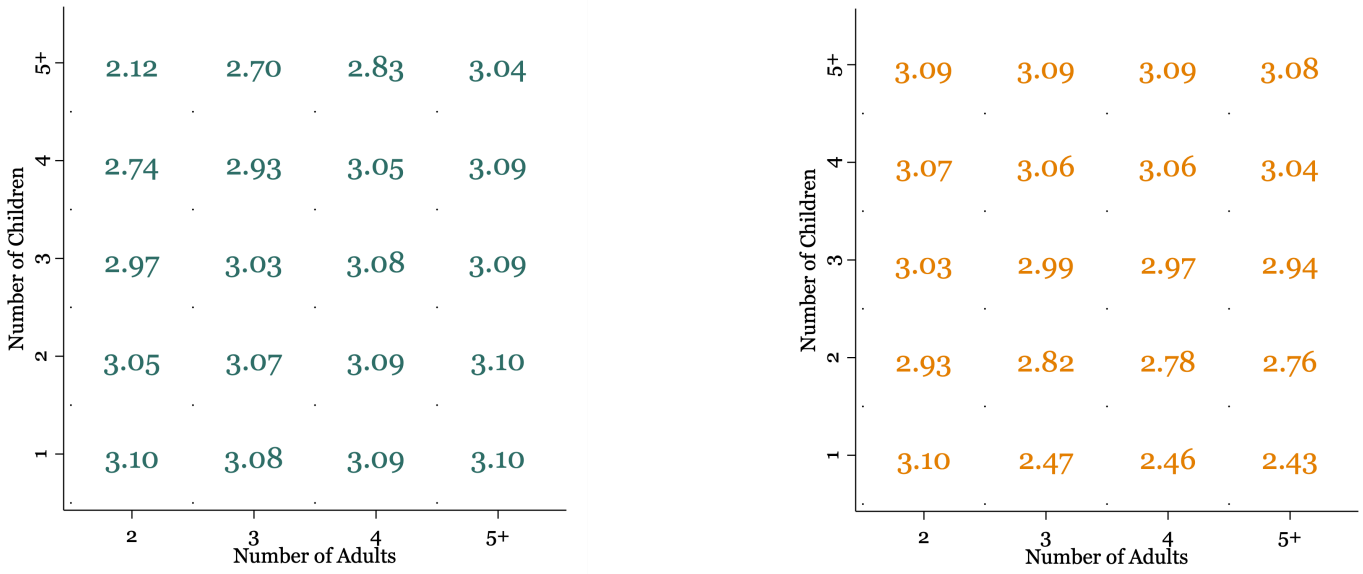
Figure A8: Model-based Poverty Lines: US$/day Adjusted for Scale Economies



(A) Bangladesh (B) Mexico

Note: The figure shows how the poverty lines for Bangladesh (Panel A) and Mexico (Panel B) should vary by number of children and adults to account for joint consumption based on our estimates. In the figure, for each household composition, the US$1.90/day poverty line is multiplied by the average level of scale economies $\widehat{s}_\tau$.

Figure A9: Model-based Poverty Lines: US$3.10/day Adjusted for Scale Economies



(A) Bangladesh (B) Mexico

Note: The figure shows how the poverty lines for Bangladesh (Panel A) and Mexico (Panel B) should vary by number of children and adults to account for joint consumption based on our estimates. In the figure, for each household composition, the US$3.10/day poverty line is multiplied by the average level of scale economies $\widehat{s}_\tau$.

C Example Indirect Utility Function

Our goal is to specify a utility function and budget share equations that are consistent with Theorem 1. We do not focus on the most general model that we can write, but rather limit our attention to a model that can be estimated. All parameters are defined as in the main text.

Suppose the indirect utility function takes the functional form of the Quadratic Almost Ideal Demand System (QUAIDS) of Banks et al. (1997):

$$V_j(p,x) = \left(\frac{P^2(p)}{x - \ln P_j^1(p)} + P_j^3(p)\right)^{-1}. \tag{A15}$$

Now assume $\ln P_j^1(\alpha + p) = \ln s_j(\alpha + p)$, $\frac{\partial P_j^3(p)}{\partial p^{k_j}} = \theta(p)$, and both $P^2(p)$ and $P_j^3(p)$ being a function of only the prices of the private goods (i.e., the sub-vector of prices of goods k for which the log Barten scale $\alpha^k = 0$).

We apply Roy's Identity to Equation (A15) at log prices $\alpha + p$ to derive the budget share equation for the private assignable good k_j:[2]

$$w^{k_j}(p, x - \ln s_j(\alpha + p)) = \frac{\partial \ln s_j(\alpha + p)}{\partial p^{k_j}} + \frac{\partial P^2(p)}{\partial p^{k_j}} \frac{(x - \ln s_j(\alpha + p))}{P^2(p)} + \theta(p) \frac{(x - \ln s_j(\alpha + p))^2}{P^2(p)}.$$

We can simplify the above expression to be:

$$w^{k_j}(p, x - \ln s_j(\alpha + p)) = a_j(\alpha + p) + b(p)(x - \ln s_j(\alpha + p)) + c(p)(x - \ln s_j(\alpha + p))^2.$$

where $a_j(\alpha + p) = \frac{\partial \ln s_j(\alpha + p)}{\partial p^{k_j}}$, $b(p) = \frac{\partial P^2(p)}{\partial p^{k_j}} \frac{1}{P^2(p)}$, and $c(p) = \frac{\theta(p)}{P^2(p)}$. The household-level Engel curve for person j's private assignable good is then:

$$W_n^{k_j}(x) = \eta_{j,n}[a_{j,n} + b(x + \ln \eta_{j,n} - \ln s_{j,n}) + c(x + \ln \eta_{j,n} - \ln s_{j,n})^2]. \tag{A16}$$

Several features of Equations (A15) and (A16) are needed so that the assumptions of Theorem 1 are satisfied. First, we assume that $\frac{\partial P_j^3(p)}{\partial p^{k_j}} = \theta(p)$ and $P^2(p)$ do not vary across people. As a result, c also does not vary across people and Assumption 3.ii is satisfied. As discussed in Corollary 1, we do not need to impose this assumption if the budget share functions are polynomials of degree $x \geq 2$ in expenditure. Nonetheless, we illustrate here how that assumption could be imposed on the QUAIDS indirect utility function. Next, we assume that $P^2(p)$ and $\theta(p)$ are nonzero so that the budget share equations are nonlinear as required by Assumption 3.iii. Finally, we allow Barten scales to only enter the utility function through P_j^1, so that preferences will not vary across household type in P^2 or P_j^3, and ultimately b and c. With this restriction, along with requiring that $P_j^1 = s_j$, the budget share functions have the same shape across household sizes as required by Assumption 4.[3] With these assumptions, it

[2] If $V_j(p,x)$ and $s_j(\alpha + p)$ satisfy Assumption 4, then Equation (A15) implies

$$V_j(\alpha + p, x) = \left(\frac{P^2(p)}{x - \ln s_j(\alpha + p)} + P_j^3(p)\right)^{-1} = V_j(p, x - \ln s_j(\alpha + p))$$

[3] Assumption 3.i, like Assumption 4, restricts how preferences vary across household type. Assumption 4 is a restriction on how the prices of shared goods (i.e., the Barten scales) enter the utility function, while Assumption 3.i says that person' type j's preferences cannot freely depend on composition of the household.

follows from Theorem 1 that $\eta_{j,n}$ and $s_{j,n}$ are identified with $s_{j,1} = 1$.

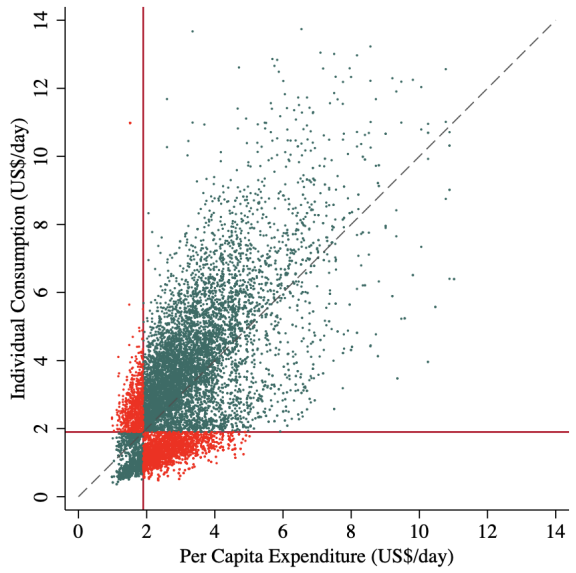
The ways in which we impose Assumptions 3.ii and 4 on Equation (A15) are similar in spirit to the "Similar Across People" (SAP) and "Similar Across Types" (SAT) of Dunbar et al. (2013), respectively. Assumption 3.ii requires us to place a restriction on how Equation (A15) varies across people, while Assumption 4 restricts the way in which the prices of shared goods enters Equation (A15). One can see that SAP and SAT imply our Assumptions 3.ii and 4 with, for example, polynomial Engel curves. Moreover, Equation (A15) is a modified version of the indirect utility function imposed by Lewbel and Pendakur (2008), who also use a QUAIDS indirect utility function. We use a less flexible functional form because we do not rely on single-person households for identification.

D Poverty Misclassification

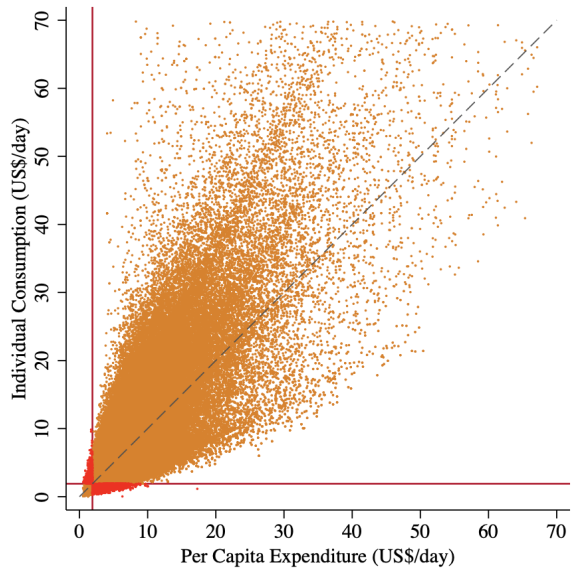
In this section, we study the extent of poverty misclassification as in Brown et al. (2022), i.e., how many people with individual consumption below the poverty line would be classified as not poor based on their per-capita expenditure, equivalent expenditure, and individual expenditure (or vice-versa). To this aim, we plot our estimates of individual consumption against household per-capita expenditure (Panels A.1 and A.2 of Figure A10), household equivalent expenditure (Panels B.1 and B.2), and individual expenditure (Panels C.1 and C.2). We partition each graph into four regions based on whether one's estimated individual consumption, per-capita expenditure, equivalent expenditure or individual expenditure is above or below the US$1.90/day poverty threshold. For individuals falling in the lower left or upper right quadrants, the two measures of poverty in each graph coincide. In other words, accounting for intra-household inequality and economies of scale does not impact their categorization as living above or below the poverty threshold. By contrast, individuals falling in the lower right quadrant would be considered poor according to the alternative measure, despite having an estimated level of individual consumption below the standard poverty line. Similarly, individuals in the upper left quadrant would be considered poor according to the alternative measures, despite having an estimated level of individual consumption above the standard poverty line.

In Bangladesh, 66.5 percent of individuals who are poor using individual consumption are misclassified as non-poor using the per-capita approach. The rate of misclassification is much higher (97.0 percent) using the OECD equivalence scale. As we do not estimate any diseconomies of scale, any individual classified as poor using individual consumption is also poor using individual expenditure. Turning to Mexico, we find similar patterns, although the misclassification rates are minimal just because the US$1.90/day threshold is very low in this context. Based on a relative poverty line (50 percent of median per-capita expenditure), we find that roughly half of poor individuals in the Mexican sample are misclassified as non-poor using the per-capita approach. We also examine misclassification in the opposite direction: individuals misclassified as poor despite having individual consumption above the poverty line. Focusing on individual expenditure, we find that approximately 1.7 and 2.6 percent of individuals who are non-poor using our consumption measure are misclassified as poor in Bangladesh and Mexico, respectively. We report total misclassification rates in Table A9 and misclassification rates separately for men, women, and children.

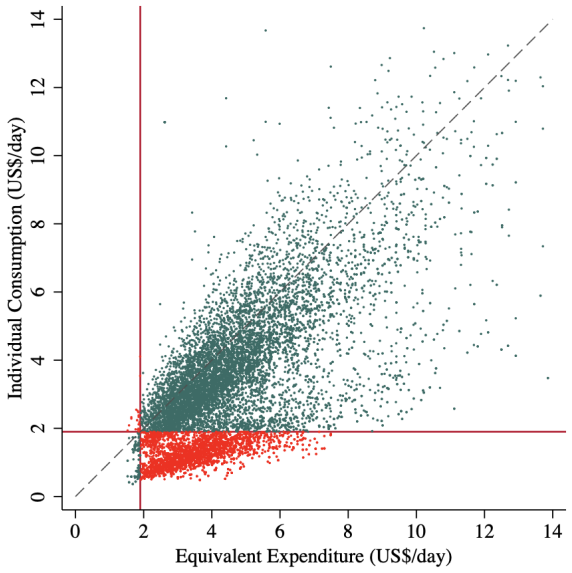
Figure A10: Per-capita Expenditure, Equivalent Expenditure, Individual Expenditure vs. Individual Consumption



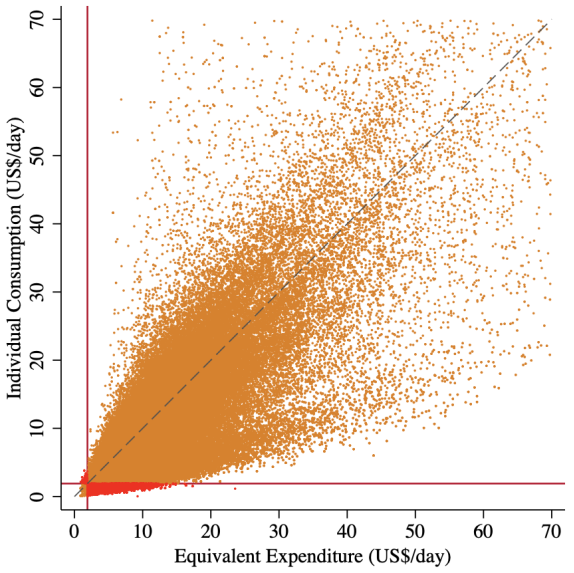
(A.1) Bangladesh: $\widehat{\eta}_{j,\tau} y/\widehat{s_\tau}$ vs. y/n



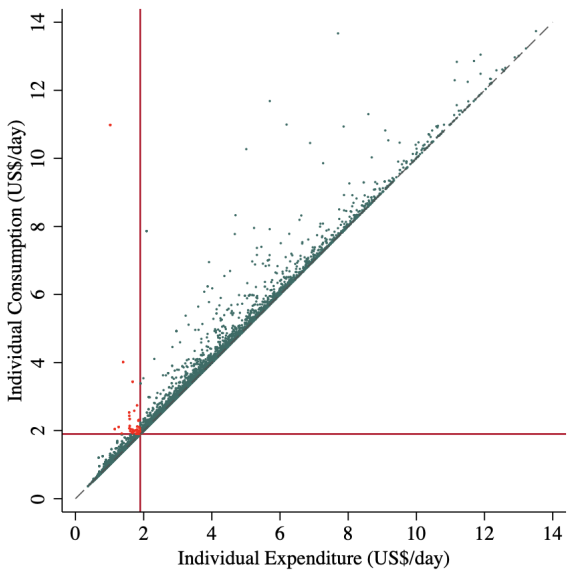
(A.2) Mexico: $\widehat{\eta}_{j,\tau} y/\widehat{s_\tau}$ vs. y/n



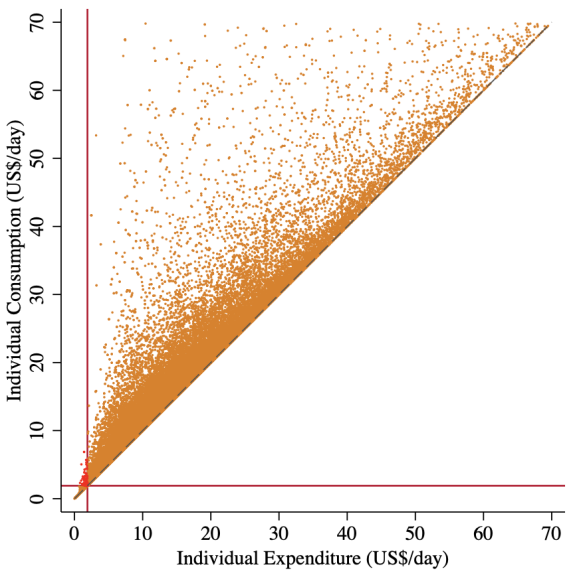
(B.1) Bangladesh: $\widehat{\eta}_{j,\tau} y/\widehat{s_\tau}$ vs. y/n_{eq}



(B.2) Mexico: $\widehat{\eta}_{j,\tau} y/\widehat{s_\tau}$ vs. y/n_{eq}



(C.1) Bangladesh: $\widehat{\eta}_{j,\tau} y/\widehat{s_\tau}$ vs. $\widehat{\eta}_{j,\tau} y$



(C.2) Mexico: $\widehat{\eta}_{j,\tau} y/\widehat{s_\tau}$ vs. $\widehat{\eta}_{j,\tau} y$

Note: The figure shows the extent of poverty misclassification, i.e., how many people with individual consumption below the poverty line would be classified as not poor based on their per-capita expenditure, equivalent expenditure, and individual expenditure (or vice-versa). Panels A.1 and A.2 report our estimates of individual consumption against household per-capita expenditure. Panels B.1 and B.2 report our estimates of individual consumption against household equivalent expenditure. Panels C.1 and C.2 report our estimates of individual consumption against individual expenditure. . Per-capita expenditure is obtained by dividing total household expenditure by the number of individuals in the household. Equivalent expenditure is calculated by dividing total household expenditure by $1 + 0.7 * (n_m + n_w - 1) + 0.5 * n_c$ where n_j gives the number of men, women, and children for $j = m, w, c$, respectively. Individual expenditure is obtained by multiplying total annual household expenditure by individual resource shares. Individual consumption is obtained by dividing individual expenditure by scale economies. In Bangladesh, only households surveyed in 2015 are included. Reference lines correspond to the US$1.90/day poverty line. Dash lines identify the 45-degree lines.

Table A9: Poverty Misclassification Rates

	Poor Based on Individual Consumption ($\widehat{\eta}_{\tau,j}y/\widehat{s}_{\tau}$)			Not Poor Based on Individual Consumption ($\widehat{\eta}_{\tau,j}y/\widehat{s}_{\tau}$)		
	Per-Capita Expenditure (y/n)	Equivalent Expenditure ($y/n_{eq.}$)	Individual Expenditure ($\widehat{\eta}_{j,\tau}y$)	Per-Capita Expenditure (y/n)	Equivalent Expenditure ($y/n_{eq.}$)	Individual Expenditure ($\widehat{\eta}_{j,\tau}y$)
	(1)	(2)	(3)	(4)	(5)	(6)
A) Bangladesh						
Children	0.728	0.983	0.000	0.007	0.000	0.079
Women	0.201	0.887	0.000	0.094	0.001	0.005
Men	0.000	0.804	0.000	0.127	0.004	0.002
Total	0.665	0.970	0.000	0.092	0.002	0.017
B) Mexico						
Children	0.707	0.901	0.000	0.009	0.002	0.044
Women	0.113	0.493	0.000	0.076	0.012	0.017
Men	0.069	0.429	0.000	0.072	0.008	0.021
Total	0.622	0.841	0.000	0.056	0.008	0.026

Note: This table shows the extent of poverty misclassification, i.e., how many people with individual consumption below the poverty line would be classified as not poor based on their per-capita expenditure, equivalent expenditure, and individual expenditure (or vice-versa). Per-capita expenditure is obtained by dividing total household expenditure by the number of individuals in the household. Individual expenditure is obtained multiplying total annual household expenditure by individual resource shares. Individual consumption is obtained by dividing individual expenditure by scale economies. In Bangladesh, only households surveyed in 2015 are included. In Bangladesh, we us a 1.90 dollar / day poverty line. In Mexico, the poverty line corresponds to the 50 percent of median per capita expenditure. We make no age adjustments to any poverty line.