Online Appendix for
Consumption Inequality Among Children: Evidence from Child Fostering in Malawi

Jacob Penglase*

July 2020

A Appendix

The Appendix is organised as follows: I first build upon the consumption results by analysing education and child labour differences across foster and non-foster children in Section A.1. In Section A.2, I examine the extent to which clothing is shared across foster and non-foster children. I also conduct several tests to determine how durable clothing is. In Section A.3, I examine whether in-kind transfers are biasing the results. Next, in Section A.4, I provide several tests of the identification assumptions. In Section A.5, I discuss how the selection of foster children into certain types of households may affect the results. I test Pareto efficiency in Section A.6. Section A.7 gives numerical examples of the ratio restrictions. I specify the full model in Section A.8. Sections A.9 and A.10 provides additional figures and tables. Finally, I present the identification theorems in Section A.11.

A.1 School Enrolment and Child Labour

To provide context to the consumption results, I examine intrahousehold inequality among foster and non-foster children along two other dimensions of welfare: education and child labour. As discussed in the main text, education and child labour are centrally linked to why

*San Diego State University. Email: jpenglase@sdsu.edu

parents foster their children. In terms of education, if the household does not live close to a school, or if the nearby school is low quality, parents may send their children to live with a relative who lives in a village with better educational access. Moreover, households may be more amenable to accepting foster children if the foster children work. For example, a household with a newborn child benefits from fostering in a young teenage girl who can care for the newborn. Alternatively, if a household has a stronger than normal harvest, they may foster in children to help with farm work. This suggests child labour may be higher among foster children.

Empirical Strategy: Unlike consumption, both school enrolment and work hours are observable at the individual level using standard household-level survey data. This facilitates a direct comparison of enrolment rates and child labour between foster and non-foster children. I begin by assigning children to two mutually exclusive groups: both biological parents absent (i.e., foster children), or at least one parent present.

I estimate the following regression for child i living in household h in region s in year t:

$$Y_{ihst} = \alpha + \gamma F_i + \pi_h + \psi_{st} + \mathbf{X_i}\delta + \epsilon_{ihst} \tag{A.1}$$

where Y_{ihst} is an indicator for school enrolment and F_i is an indicator variable equal to one if the child is fostered. In other specifications, Y_{ihst} is hours worked. Since this variable is censored at zero, I use a Tobit model and the system is estimated via maximum likelihood. $\mathbf{X_i}$ is a vector of individual characteristics, such as child age and gender. The parameter of interest is γ, which captures the effect of the absence of a child's parents on the various outcomes of interest. In some specifications I include household fixed effects to control for any unobserved heterogeneity that does not vary over time. Household fixed effects allow for the direct examination of unequal treatment between foster and non-foster children, as I am relying only on within-household variation. Lastly, I include region-year fixed effects to account for any region specific year effects that are common across foster status and households. I cluster standard errors at the region-year level.

Existing work has found orphaned-foster children have lower school enrolment (Case et al., 2004; Ainsworth and Filmer, 2006). I therefore modify the above estimation to account for orphan status in order to examine whether a similar pattern emerges here. I now assign children into four mutually exclusive groups: non-orphaned non-foster; orphaned non-foster; non-orphaned foster; orphaned foster. I estimate the following specification:

$$Y_{ihst} = \alpha + \gamma_1 O_i + \gamma_2 F_i + \gamma_3 (O_i \times F_i) + \pi_h + \psi_{st} + \mathbf{X_i}\delta + \epsilon_{ihst} \tag{A.2}$$

where F_i and O_i are indicators for foster and orphan status respectively. The parameters of interest are now γ_1, γ_2 and γ_3, which capture the differential effects of the child's foster and orphan status on school enrolment or child labour. The omitted category is non-orphaned children with at least one biological parent present. I again use the Malawi Integrated Households Survey (IHS3 and IHS4) and the Malawi Integrated Panel Survey. Descriptive statistics are presented in Table A22 in the Appendix.

Table A1: School Enrollment by Foster Status

	LPM			Probit
	(1)	(2)	(3)	(4)
Foster Child	-0.022***	-0.013*	-0.052***	-0.084**
	(0.006)	(0.007)	(0.014)	(0.041)
Sample Size	35,198	35,198	35,198	35,198
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls		Yes	Yes	Yes
Household Fixed Effects			Yes	

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all children age 6 to 14. The omitted fostering category are children with at least one biological parent present. Robust standard errors. Columns 1-3 provide estimates for a linear probability model. Column 4 presents marginal effects for a probit specification. Individual controls include age fixed effects, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Results: I begin by analysing the difference in school enrolment rates between foster and non-foster children. I estimate Equation (A.1) and present the results in Table A1. The coefficient of interest γ describes the difference in treatment for foster and non-foster children. Column (1) provides an estimate of differences in means by foster status, controlling for child age and gender. This specification ignores any household characteristics that may be associated with both school enrolment rates and the types of households that foster in children. Columns (2) and (3) attempt to uncover evidence of intrahousehold discrimination of foster children. In column (2), I account for observable household characteristics, including the education, age, and gender of the household head, household composition measures, and log per capita household expenditure. In column (3), I include household fixed effects, which accounts for any unobservable household characteristics that do not vary across time. The results provide evidence that foster children are enrolled in school at lower rates than non-foster children.

I next examine whether this pattern is driven by orphaned-foster children. I estimate Equation (A.2) with four foster categories that account for orphanhood. The results are presented in Table A2. Columns (1) to (3) present results from a linear probability model with increasing controls moving left to right. The findings largely show that orphaned foster children are driving the results, as these children are less likely to be enrolled in school than non-foster children. The preferred specification is provided in column (4), where the model is estimated via a probit model. The displayed parameter gives the marginal effects, and suggest that orphaned foster children are 15.1 percentage points less likely to be enrolled in school than non-orphaned non-foster children.

Table A2: School Enrollment by Foster Status (Detailed Categories)

	LPM			Probit
	(1)	(2)	(3)	(4)
Fostering Categories				
Non-Orphaned Foster	-0.015**	-0.006	-0.052***	-0.049
	(0.008)	(0.008)	(0.018)	(0.049)
Orphaned Foster	-0.037***	-0.026**	-0.045**	-0.151***
	(0.010)	(0.010)	(0.018)	(0.056)
Orphaned Non-Foster	-0.020**	-0.011	0.012	-0.054
	(0.010)	(0.010)	(0.024)	(0.055)
Sample Size	35,198	35,198	35,198	35,198
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls		Yes	Yes	Yes
Household Fixed Effects			Yes	

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all children age 6 to 14. The omitted fostering category are non-orphaned children with at least one biological parent present. Robust standard errors. Columns 1-3 provide estimates for a linear probability model. Column 4 presents marginal effects for a probit specification. Individual controls include age fixed effects, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

Table A3 provides the child labour results. In columns (1) and (2), I examine the relationship between foster status and hours worked doing chores,[1] while columns (3) and (4) focus on hours worked for a household farm, household enterprise, ganyu labour, apprenticeships, or wage work outside the household in the previous week. I add controls moving from left to right. The results provide little evidence that work around the house differs substantially between foster and non-foster children, which is contrary to what the theoretical literature

[1] Chores include fetching wood and fetching water.

suggests (Serra, 2009), but consistent with recent empirical work by Beck et al. (2015). This lack of any effect in column (2) is partially due to the limited definition of chores (only fetching wood and water), and possible measurement error in the data, as parents may be unwilling to reveal that their children work. Table A4 accounts for orphanhood when examining the effect of foster status on child labour. The results provide some evidence that orphaned-foster children work more than non-orphaned non-foster children. The results in column (2), which rely only in within household variation, suggest that orphaned-foster children spend 0.59 more hours per week. The results for work outside the household counterintuitively suggest non-orphaned foster children work less than non-orphaned non-foster children.

Table A3: Weekly Hours Worked by Fostering Status

	Chores		Work Outside HH	
	(1)	(2)	(3)	(4)
Foster Child	0.601***	0.182	0.193	-0.711
	(0.202)	(0.227)	(0.412)	(0.451)
Sample Size	35,198	35,198	35,198	35,198
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls	Yes	Yes	Yes	Yes
Household Fixed Effects		Yes		Yes

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all children age 6 to 14. The omitted fostering category are children with both biological parents present. Standard errors are clustered at the region-year level. Individual controls include age, age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

A.2 Is Clothing a Private Good?

Sharing of Purchased Clothing The model requires that clothing is not shared across person types. This assumption means that foster children cannot share clothing with non-foster children, and vice versa.[2] Hand-me-down clothing is a separate issue that is discussed later. While this assumption may at first seem worrisome, there are several reasons it is not of too great a concern. First, clothing includes shoes and school uniforms, both of which are difficult

[2] If I instead studied inequality between boys and girls, this issue of sharing becomes significantly less problematic as clothing is often gender-specific. Thus, any concerns about sharing are partly due to the nature of child fostering, and less so with the identification method in general.

Table A4: Weekly Hours Worked by Fostering Status (Detailed Categories)

	Chores		Work Outside HH	
	(1)	(2)	(3)	(4)
Fostering Categories				
Non-Orphaned Foster	0.530**	0.024	-0.098	-1.155**
	(0.245)	(0.272)	(0.499)	(0.535)
Orphaned Foster	0.878***	0.594*	0.789	-0.020
	(0.315)	(0.324)	(0.646)	(0.665)
Orphaned Non-Foster	0.544*	0.386	0.445	0.028
	(0.309)	(0.322)	(0.600)	(0.623)
Sample Size	35,198	35,198	35,198	35,198
Region-Year Fixed Effects	Yes	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Household Controls	Yes	Yes	Yes	Yes
Household Fixed Effects		Yes		Yes

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all children age 6 to 14. The omitted fostering category are non-orphaned children with at least one parent present. Standard errors are clustered at the region-year level. Individual controls include age, age^2, and gender. Household controls include the number of male and female siblings age 0-6 and 7-14, the number of adult men and women, log household expenditure, and demographic characteristics of the household head. * p<0.1, ** p<0.05, *** p<0.01

to share. Second, foster children are typically different ages than the non-foster children within the household; Fostering is often used to balance the demographic structure of the household in order to maximize household production (Akresh, 2009). As a result, it is somewhat rare to have a foster and non-foster child of the same age and gender in a given household.

To examine the merit of this assumption, I drop all households with both foster and non-foster children in any of the following age groups: 0-3, 4-7, 8-11, and 12-14, and re-estimate the model. Since foster and non-foster children in different age groups are unlikely to share clothing, I can confidently assume clothing is private in this restricted sample. Table A5 presents the results. In the age-restricted sample, resource shares for foster children are not statistically different from the unrestricted results, and are quite similar. There is also not any consistent increase in non-foster child resource shares with the age restrictions.

To examine this in a different way, I estimate children's clothing Engel curves. Specifically, I allow the Engel curve to vary with the presence of multiple types of children in the same age group. If sharing of clothes between foster and non-foster children were present, we would expect the coefficients on these variables to be negative. The results are presented in Table A6. In column (1), I include indicators for four different age groups. In column (2) I include an indicator for having both a foster and non-foster child in any age group. Sharing of clothing

Table A5: Determinants of Resource Shares: Age-Restricted Sample

	Non-Foster Children		Foster Children	
	Main Results	Age-Restricted Sample	Main Results	Age-Restricted Sample
	(1a)	(2a)	(1b)	(2b)
Household Type Indicators				
2 Non-Foster 0 Foster	0.251***	0.245***		
	(0.0484)	(0.0468)		
1 Non-Foster 1 Foster	0.151***	0.141***	0.171	0.160
	(0.0313)	(0.0300)	(0.0355)	(0.0352)
0 Non-Foster 2 Foster			0.301***	0.297***
			(0.0637)	(0.0638)
3 Non-Foster 0 Foster	0.285***	0.280***		
	(0.0548)	(0.0529)		
2 Non-Foster 1 Foster	0.201	0.191277082	0.154***	0.133***
	(0.0381)	0.036788959	(0.0342)	(0.0316)
1 Non-Foster 2 Foster	0.148***	0.117***	0.241	0.232
	(0.0381)	(0.0383)	(0.0530)	(0.0521)
0 Non-Foster 3 Foster			0.363***	0.359***
			(0.0839)	(0.0846)
Covariates				
Average Age non-Foster	1.435**	1.706***	-0.227	0.219
	(0.586)	(0.591)	(0.771)	(0.637)
Average Age non-Foster^2	-0.0811*	-0.0965**	0.0186	-0.0225
	(0.0418)	(0.0422)	(0.0546)	(0.0462)
Average Age Foster	-0.545	-0.455	2.428	2.482*
	(1.888)	(2.444)	(1.609)	(1.328)
Average Age Foster^2	0.0435	0.0424	-0.120	-0.120
	(0.107)	(0.133)	(0.102)	(0.0860)
Proportion Non-Foster Female	-0.0146	-0.0135	-0.00524	-0.00941
	(0.0112)	(0.0109)	(0.0174)	(0.0158)
Proportion Foster Female	0.00794	0.0153	-0.0295	-0.0255
	(0.0268)	(0.0264)	(0.0333)	(0.0248)
Rural	0.00277	0.00647	-0.000770	0.00132
	0.00277	0.00647	-0.000770	0.00132
Matrilineal Village	0.00777	0.00778	0.0180	0.0181*
	(0.00948)	(0.00917)	(0.0118)	(0.00973)
Proportion of Fostered Orphaned	0.0265	0.0254	-0.0190	-0.0112
	(0.0259)	(0.0237)	(0.0289)	(0.0245)
Sample Size	17,203	16,766	17,203	16,766
Log Likelihood	150,467	146,438	150,467	146,438

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Estimates for certain household types and preferences factors are omitted for conciseness. The parameters on the household type indicators are not per child, but the total allocation to all foster or non-foster children within the household. The Age-Restricted Sample drops households with both foster and non-foster children in any of the following age groups: 0-3, 4-7, 8-11, and 12-14. * p<0.1, ** p<0.05, *** p<0.01

does not appear to matter for children age 4 and above. However, households that have both a foster and non-foster child age 0 to 3 do spend less on clothing. While this is not ideal, only 33 out of 17,203 households fit this description. Overall, the results do not suggest that sharing is having a large effect on the model estimates.

Table A6: Child Clothing Expenditures by Child Age Composition

Dependent Varaible: Child Clothing Budget Shares	(1)	(2)
Foster and Non-Foster Age 0-3	-0.005**	
	(0.002)	
Foster and Non-Foster Age 4-7	0.000	
	(0.002)	
Foster and Non-Foster Age 8-11	-0.000	
	(0.002)	
Foster and Non-Foster Age 12-14	0.002	
	(0.002)	
Foster and Non-Foster in Same Age Group		0.000
		(0.001)
Sample Size	17,203	17,203
Region Fixed Effects	Yes	Yes
Year Fixed Effects	Yes	Yes

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. Column (1) includes indicators for whether at least one foster or non-foster child in a given age group. Column (2) includes a single indicator equal to one if any age group has both a foster and non-foster child. Additional controls include log household expenditure and the number of each person-type within the household. * p<0.1, ** p<0.05, *** p<0.01

Hand-Me-Down Clothing Given the age difference of foster and non-foster children, one might further be concerned about hand-me-down clothing. Specifically, clothing may be better characterized as a semi-durable good rather than non-durable. In the model and estimation, I define children's clothing expenditures to be the amount the household spends on children's clothing within the past year (i.e., purchased clothing). For the identification assumptions to be violated, the relationship between hand-me-down clothing and purchased clothing would have to be different in composite and one-child-type households in such a way that is correlated with total expenditure. To see why, first note that preferences for purchased clothing do not have to be identical across composite and one-child-type households. Preferences just have to be similar; Preferences for purchased clothing can differ across composite and one-child-type households in the intercept preference parameter δ_s^t, but not the slope parameter β^t. As a result, if foster or non-foster children consume a large amount of hand-me-down clothing, then they would have a lower δ_s^t in the Engel curve for purchased clothing. This is allowed. A violation could occur if the existence of hand-me-down clothing affects the marginal propensity for foster or non-foster children to consume purchased clothing (i.e., β^t).

To examine this empirically, I estimate household-level children's clothing Engel curves. The test will rely on the following two assumptions; first, hand-me-down clothing only exists within gender. That is, younger boys can consume clothing that was once worn by older boys, but younger boys cannot consume clothing that was once worn by older girls. Moreover, I make the strong assumption that boys and girls have identical preferences for clothing. Then, conditional on household size, we would expect clothing expenditure to be less in households with same-gender children if hand-me-down clothing were present.

Let $Boys\_HH = 1$ if the household has more male than female children, and 0 otherwise. Let $Girls\_HH = 1$ if the household has more female than male children, and 0 otherwise. The omitted category will be households with an equal number of male and female children, where hand-me-down clothing is potentially less common. To test this hypothesis, I estimate the following regression:

$$W_s^{clothing} = \beta_0 + \beta_1 Boys\_HH + \beta_2 Girls\_HH + \beta_3 \ln y + \mathbf{X_s}\gamma + \epsilon_s \tag{A.3}$$

where $W_s^{clothing}$ is purchased children's clothing in a household of type s, $\mathbf{X_s}$ is a vector of household characteristics that includes year and region fixed effects, as well as the number of men, women, and children in the household. Log household expenditure is given by $\ln y$. If hand-me-down clothing were present, we would expect $\beta_1 < 0$ and $\beta_2 < 0$, as (unobserved) hand-me-down clothing would substitute for (observed) purchased clothing.

However, as discussed above, this is permissible as long as the extent of hand-me-done clothing consumption is independent of household expenditure. I therefore interact the gender composition of the household's children with log household expenditure.

$$\begin{aligned} W_s^{clothing} =& \beta_0 + \beta_1 Boys\_HH + \beta_2 Girls\_HH + \beta_3 \ln y \\ &+ \beta_4 (Boys\_HH * \ln y) + \beta_5 (Girls\_HH * \ln y) + \mathbf{X_s}\gamma + \epsilon_s \end{aligned} \tag{A.4}$$

If hand-me-down clothing were changing the marginal propensity of child clothing consumption, we would expect $\beta_4 \neq 0$ and $\beta_5 \neq 0$.

The results are provided in Table A7. Column (1) presents the results from Equation (A.3). The results suggest that households with more girls than boys spend more on clothing than those with an equal number of boys and girls, all else equal. No difference is seen between households with a majority boys and those with an equal share. This suggests that either hand-me-down clothing is present, at least for girls, or that boys and girls have different preferences for clothing. I am unable to distinguish between these explanations with the given data.

Table A7: Child Clothing Expenditures by Gender Composition

Dependent Varaible: Child Clothing Budget Shares	(1)	(2)
Boys_HH	-0.001	-0.008
	(0.001)	(0.009)
Girls_HH	-0.001**	-0.009
	(0.001)	(0.009)
Log Household Expenditure	0.004***	0.003***
	(0.000)	(0.001)
Boys_HH × Log Household Expenditure		0.001
		(0.001)
Girls_HH × Log Household Expenditure		0.001
		(0.001)
Sample Size	17,203	17,203
Covariates	Yes	Yes
Child Number Fixed Effects	Yes	Yes
Region Fixed Effects	Yes	Yes
Year Fixed Effects	Yes	Yes

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. Boys_HH is an indicator equal to one if the majority of children in the household are boys. Girls_HH is the equivalent for girls. Additional covariates include the number of men and women in the household, the share of children who are fostered, and whether the household lives in an urban or rural area. * p<0.1, ** p<0.05, *** p<0.01

In column (2), I present results from Equation (A.4). Here I interact the gender composition of the household with log expenditure. The purpose of this exercise is to determine if hand-me-down clothing enters the model in such a way that may bias the results. Encouragingly, the results suggest that the extent of hand-me-down clothing consumption does not vary by household expenditure.

Overall, hand-me-down clothing is not conclusively absent. A natural question then is to what extent this may bias the results. This question relates more generally to the validity of using clothing as a private assignable good. In response, I would cite two recent papers that take different approaches to validate the use of clothing as a means to identify resource shares.

Recent work by Bargain et al. (2018) and Lechene et al. (2019) provide evidence that hand-me-down clothing is not likely to significantly bias the results. Bargain et al. (2018) use a data set containing observable individual consumption to show that clothing works extremely well as an assignable good to identify resource shares in the framework of a collective household model. Lechene et al. (2019) demonstrate that using clothing and food result in similar resource share estimates, and food clearly has no durable elements.

A.3 In-Kind Transfers

One potential concern is that foster children are receiving clothing and other goods from their biological parents. This may lead to downwardly biased resource share estimates for foster children. I examine the degree to which both in-kind transfers and remittances may be affecting foster child demand for clothing. I use self-reported measures of in-kind transfers (i.e., non-monetary transfers received from other households) and remittances.[3]

To determine if these transfers affect clothing consumption, I regress child clothing budget shares on log in-kind transfers and remittances. Moreover, I allow the relationship between transfers and child clothing budget shares to vary with the presence of foster children. I control for several household characteristics, such as log household expenditure. If foster children were receiving clothing from other households, we would expect their demand for clothing to be decreasing in the value of in-kind transfers. This proves not to be the case for both in-kind transfers and remittances. The results, presented in Table A8, show that in-kind transfers have no effect on clothing demand, nor does this relationship vary by the presence of a foster child in the household. The same finding holds for remittances.

Table A8: Child Clothing Expenditures by Transfers

Dependent Varaible: Child Clothing Budget Shares	In-Kind Transfers		Remittances	
	(1)	(2)	(3)	(4)
Log In-Kind Transfers	0.00002	0.00001		
	(0.00004)	(0.00005)		
Foster Household		-0.00140***		-0.00176***
		(0.00049)		(0.00040)
Foster Household × Log In-Kind Transfers		0.00005		
		(0.00009)		
Log Remittances			0.00002	0.00003
			(0.00002)	(0.00003)
Foster Household × Log Remittances				0.00004
				(0.00006)
Sample Size	17,203	17,203	17,203	17,203
Region Fixed Effects	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. Remittances are monetary transfers from non-residents to the household. In-kind transfers are non-monetary remittances. Columns (2) and (4) allow the effect of in-kind transfers and remittances to vary with the presence of a foster child within the household. Additional controls include log household expenditure and child age. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

[3] Unfortunately I do not observe the types of goods received, only their estimated value.

I next include in-kind transfers as a covariate in the resource share functions. If this covariate is negatively associated with foster child resource shares, that may suggest measurement error in the foster child resource share estimates. These results are presented in Table A9 in column (2). In a different specification, I restrict the sample to households that received any in-kind transfers. These results are presented in column (3). In both specifications, in-kind transfers do not seem to be affecting the results.

Table A9: Determinants of Foster Child Treatment: In-Kind Transfers

	Main Results (1)	In-Kind Transfers in Resource Share Function (2)	HH Received In-Kind Transfers (3)
Household Type Indicators			
1 Non-Foster 1 Foster	0.171	0.168	0.170
	(0.0355)	(0.0357)	(0.0400)
0 Non-Foster 2 Foster	0.301***	0.297***	0.286***
	(0.0637)	(0.0637)	(0.0667)
2 Non-Foster 1 Foster	0.154***	0.152***	0.151***
	(0.0342)	(0.0342)	(0.0364)
1 Non-Foster 2 Foster	0.241	0.238	0.230
	(0.0530)	(0.0529)	(0.0600)
0 Non-Foster 3 Foster	0.363***	0.358***	0.340***
	(0.0839)	(0.0837)	(0.0878)
Covariates			
In-Kind/Expenditure		0.235	
		(0.612)	
Average Age non-Foster	-0.227	-0.220	-0.176
	(0.771)	(0.761)	(0.837)
Average Age non-Foster^2	0.0186	0.0179	0.0134
	(0.0546)	(0.0539)	(0.0586)
Average Age Foster	2.428	2.378*	2.152
	(1.609)	(1.294)	(1.346)
Average Age Foster^2	-0.120	-0.118	-0.104
	(0.102)	(0.0875)	(0.0960)
Proportion Non-Foster Female	-0.00524	-0.00513	0.00281
	(0.0174)	(0.0172)	(0.0191)
Proportion Foster Female	-0.0295	-0.0290	-0.0294
	(0.0333)	(0.0324)	(0.0379)
Rural	-0.000770	-0.000656	-0.00231
	-0.000770	-0.000656	-0.00231
Matrilineal Village	0.0180	0.0179	0.0148
	(0.0118)	(0.0114)	(0.0128)
Proportion of Fostered Orphaned	-0.0190	-0.0180	-0.0159
	(0.0289)	(0.0287)	(0.0341)
Sample Size	17,203	17,203	16,927
Log Likelihood	150,467	150,470	148,085

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Estimates for certain household types and preferences factors are omitted for conciseness. The parameters on the household type indicators are not per child, but the total allocation to all foster or non-foster children within the household. * p<0.1, ** p<0.05, *** p<0.01

A.4 Are the Restrictions Valid?

Are One-Child-Type and Composite Households Similar? For the main estimation results, I impose some similarity in the clothing Engel curves across households with only foster or non-foster children, and those with both. I analyse the validity of this assumption indirectly. I first ask, are one-child-type and composite households similar? To answer this question, I compute sample means of different household characteristics for one-child-type and composite households. If households with only foster (or non-foster) children differ from composite households over observable characteristics, that may suggest they differ in unobservable ways, which may limit the validity of the restrictions. Table A10 presents sample means for several household characteristics by the different household compositions.

Table A10: Sample Means by Household Composition

	One-Child-Type		Composite
	Only Non-Foster (1)	Only Foster (2)	(3)
Men	1.332	1.485	1.551
Women	1.296	1.413	1.531
Non-Foster	2.314		1.712
Foster		1.655	1.215
Log Real Total Expenditures	11.941	12.015	12.029
Year=2010	0.429	0.408	0.407
Year=2013	0.143	0.158	0.185
Foster Child Age		9.275	9.444
Non-Foster Child Age	5.846		6.476
Proportion Orphaned of Foster Children		0.300	0.369
Proportion Female of non-Foster	0.502		0.493
Proportion Female of Foster		0.552	0.557
Average Age Women	29.903	49.342	32.511
Average Age Men	32.668	42.860	33.330
Average Education Women	1.042	0.774	1.173
Average Education Men	1.252	1.129	1.405
Share Women Age 15-18	0.071	0.096	0.096
Share Men Age 15-18	0.102	0.200	0.150
Rural	0.811	0.828	0.722
Matrilineal Village	0.543	0.526	0.514
Sample Size	14,213	1,549	1,441

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. One-child-type households contain either only non-foster children or only foster children. Composite households contain both foster and non-foster children.

The results are mostly positive; encouragingly, foster and non-foster child characteristics, such as age and gender, do not seem to vary much between one-child-type and composite households. Unfortunately, adult characteristics, such as age and education, differ across one-child-type foster households and the composite households. The underlying reason for this is that households that have only foster children tend to be households where the foster children are cared for by grandparents, while in composite households foster children are typically cared for by their aunt and uncle, who have their own non-foster (biological) children. Table A11 presents the percentage of foster children cared for by different relatives in households with only foster children, and in households with both foster and non-foster children.

Table A11: Distribution of Foster Caretakers by Household Composition

	All Foster Households	Households With Both Foster and Non-Foster Children	Households With Only Foster Children
	(1)	(2)	(3)
Foster Caretaker			
Grandparent(s) and Uncle/Aunt	23.45	30.78	19.05
Uncle/Aunt Only	10.76	18.67	6.01
Grandparent(s) Only	43.02	10.51	62.56
Adopted	12.85	20.94	7.98
Other*	9.91	19.09	4.39
Sample Size	3,168	1,189	1,979

Notes: Malawi Integrated Household Panel Survey 2016. The sample includes all foster children. *Other includes children living with an older sibling, other relatives, or other non-relatives.

Since one-child-type and composite households do seem to differ in some ways across the entire sample, I next examine if there is overlap among subsamples of the different household types. To do this, I select two subsamples of one-child-type households (foster only and non-foster only) that are most similar to the composite households using a propensity score matching procedure.[4] The results are presented in Table A12. Columns (1) and (2) compare households with only non-foster children to households with both non-foster and foster children. I do the same for foster one-child-type households in columns (3) and (4). None of the estimated means are statistically different across the matched subsamples. Then since the model does allow for observable heterogeneity in the resource share parameters, concerns regarding potential violations due to differences in composite and one-child-type households are likely minimal.

[4] I use nearest neighbour propensity score matching, where households are selected based on the covariates listed in Table A18. In comparing non-foster one-child-type households with composite households, I drop one-child-type households and match them with the full sample of composite households. When I compare foster one-child-type households with composite households, I select a subsample of similar one-child-type foster households and composite households.

Table A12: Sample Means by Household Composition

	Matched Sample		Matched Sample	
	Non-Foster Only (1)	Composite (2)	Foster Only (3)	Composite (4)
Men	1.557	1.551	1.615	1.613
Women	1.476	1.531	1.641	1.654
Non-Foster	1.691	1.712		1.608
Foster		1.215	1.361	1.359
Log Real Total Expenditures	12.059	12.029	12.067	12.098
Year=2010	0.399	0.407	0.396	0.396
Year=2013	0.144	0.185	0.182	0.184
Foster Child Age		9.444	8.952	8.190
Non-Foster Child Age	6.483	6.476		8.856
Proportion Orphaned of Foster Children		0.369	0.307	0.285
Proportion Female of non-Foster	0.490	0.493		0.503
Proportion Female of Foster		0.557	0.537	0.542
Average Age Women	32.388	32.511	38.391	39.070
Average Age Men	33.111	33.330	36.163	37.303
Average Education Women	1.205	1.173	1.033	1.030
Average Education Men	1.426	1.405	1.302	1.290
Share Women Age 15-18	0.091	0.096	0.136	0.138
Share Men Age 15-18	0.145	0.150	0.201	0.195
Rural	0.701	0.722	0.755	0.753
Matrilineal Village	0.514	0.514	0.527	0.495
Sample Size	1,411	1,411	732	732

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. One-child-type households contain either only non-foster children or only foster children. Composite households contain both foster and non-foster children. Matched samples are selected using propensity score matching. In total, there are 1,411 composite households that are matched with a corresponding one-child-type non-foster household. There are 1549 households with only foster children, and out of those households I select 732 to match with the most similar composite households. None of the variables are statistically different at the 5% level across one-child-type and composite households.

Are Ratio Restrictions 1 and 2 Valid? To test the validity of the ratio restrictions, I estimate the model following the identification approach developed in Section **??**. Using this method, I do not need to assume any relationship in resource shares across household types. I can then test whether or not the estimated resource shares are consistent with Ratio Restriction's 1 and 2. Specifically, I test the following null hypotheses which are assumed to hold by Restriction 1: $\eta^a_{s_{a0}} = \frac{\eta^a_{s_{a+1,0}} \eta^a_{s_{ab}}}{\eta^a_{s_{a+1,b}}}$ and $\eta^b_{s_{0b}} = \frac{\eta^b_{s_{0,b+1}} \eta^b_{s_{ab}}}{\eta^b_{s_{a,b+1}}}$ for households with one to four foster and non-foster children; and Restriction 2: $\eta^b_{11} = \frac{\eta^a_{11}\eta^b_{01}}{\eta^a_{10}}$. Overall, I consistently fail to reject the hypothesis that the restrictions hold. While the resource shares are not estimated that precisely and therefore the hypotheses are difficult to reject, the restrictions are still largely consistent with the estimated resource shares. The parameter estimates used for these tests are presented in Table A13.

Table A13: Determinants of Resource Shares: Estimation with SAT Restriction

	Non-Foster Children	Foster Children
	(1)	(2)
	NLSUR	NLSUR
1 non-Foster 0 Foster	0.181***	
	(0.0429)	
2 non-Foster 0 Foster	0.247***	
	(0.0525)	
3 non-Foster 0 Foster	0.283***	
	(0.0596)	
4 non-Foster 0 Foster	0.317***	
	(0.0678)	
0 non-Foster 1 Foster	.	0.207***
	.	(0.0491)
1 non-Foster 1 Foster	0.113**	0.200***
	(0.0511)	(0.0580)
2 non-Foster 1 Foster	0.169**	0.175***
	(0.0686)	(0.0616)
3 non-Foster 1 Foster	0.226***	0.154***
	(0.0735)	(0.0587)
0 non-Foster 2 Foster	.	0.286***
	.	(0.0673)
1 non-Foster 2 Foster	0.0855	0.283***
	(0.0557)	(0.0674)
2 non-Foster 2 Foster	0.164**	0.260***
	(0.0742)	(0.0733)
0 non-Foster 3 Foster	.	0.346***
	.	(0.0881)
1 non-Foster 3 Foster	0.0842*	0.337***
	(0.0493)	(0.0855)
0 non-Foster 4 Foster		0.411***
		(0.110)
Number of Men	-0.00146	-0.00263
	(0.00685)	(0.00706)
Number of Women	-0.00307	-0.00404
	(0.00922)	(0.00872)
Sample Size	17,203	
Log Likelihood	150,455	

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Coefficients on the covariates (age, education, etc.) are omitted for conciseness. Several household types are dropped from the sample due to too few observations. The parameter estimates are not per child, but rather the total resources allocated to foster or non-foster children. Restrictions 1 and 2 are not imposed in the estimation. * p<0.1, ** p<0.05, *** p<0.01

Lastly, it is useful to note that in principle, these restrictions are testable with additional data. If I observed assignable goods for foster and non-foster children, I could analyse how preferences vary across household types. I leave that for future work.

A.5 Is There Selection Bias?

Foster and non-foster children are not randomly assigned into households. The decision to foster one's children, and the decision to receive a foster child is a complicated process. Furthermore, households that decide to accept a foster child may be different from households without foster children in unobservable ways that are correlated with the treatment of foster and non-foster children. For example, a household with non-foster children that refuses to take in a foster child may do so because they prefer to devote more resources to their own biological children.

In this paper, I do not model the fostering decision as others have done (Ainsworth, 1995; Akresh, 2009; Serra, 2009), but instead analyse the material well-being of children conditional on being in a given household. In other words, I do not analyse the causal effect of living in a foster household on child treatment. I am more interested in a descriptive analyses of the well-being of children currently being fostered. Nevertheless, I briefly examine whether or not selection of children into different household types affects foster and non-foster child treatment. The primary concern is that there is a subset of one-child-type, non-foster households who are driving the results, and that these households are different in unobservable ways from the composite households. If this were true, imposing any similarity between these different household types may be problematic.

To determine the severity of this concern, I attempt to drop these "problem" households. I conduct a matching exercise using covariates included in the model to select a subsample of one-child-type, non-foster households that are most similar to the composite households using nearest neighbour propensity score matching.[5] The motivation behind this procedure is to improve the common support of the different types of households. I estimate the model on the subsample of one-child-type households and compare these results to the main estimation results. The results are presented in Table A14. Columns (1) and (2) display the predicted per non-foster child resource shares for a reference household. Column (1) presents the results for the full sample, while column (2) does the same for the restricted sample. Overall, there are no statistical differences between the results, suggesting that for non-foster children, selection bias is not a concern.[6]

[5] See Table A12 columns (1) and (2).

[6] I lack a sufficient number of households to proceed with a similar analysis of one-child-type foster households.

Table A14: Predicted Resource Shares: Households with Only Non-Foster Children

Household Type	Main Results (1)	Restricted Sample (2)
1 non-Foster 0 Foster	0.197	0.184
	(0.0415)	(0.0762)
2 non-Foster 0 Foster	0.130	0.113
	(0.0248)	(0.0449)
3 non-Foster 0 Foster	0.097	0.085
	(0.0183)	(0.0332)
4 non-Foster 0 Foster	0.081	0.065
	(0.0156)	(0.0272)
Sample Size	17,203	1,441
Log Likelihood	150,467	12,529

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The full sample includes all households with 1-4 men and women, and 1-4 children. The restricted sample is selected using nearest neighbor propensity score matching. In total, there are 1,441 composite households which are matched with one-child-type non-foster households. These matched households comprise the restricted sample. Robust standard errors in parentheses. The predicted resource shares are per-child. * p<0.1, ** p<0.05, *** p<0.01

A.6 Testing Pareto Efficiency

One of the central assumptions of the collective model is that the ultimate allocation of resources is Pareto efficient. That is, there is no way of reallocating goods in such a way that makes one person better off, without making someone else worse off. Pareto efficiency is a testable assumption. Past work has tested it in the context of Malawi (Dunbar et al., 2013) and has failed to reject the assumption. But the test was conducted on a sample of households from 2004, and only on nuclear households. Encouragingly, recent work by Rangel and Thomas (2019) tests Pareto efficiency in complex households in Burkina Faso and fails to reject the assumption.

I use the Malawian data to conduct several tests of Pareto efficiency as well. To do so, I rely on distribution factors. Distribution factors are variables that affect the relative standing of each person in the household, but not each person's preferences. Stated differently, these are variables that enter the Pareto weights, but not each person's individual utility function. Examples include divorce laws or the share of assets owned by a particular person in the household. In this context, I rely on the kinship system in the village the household resides (patrilineal vs. matrilineal), and education differences across adult men and women.

I follow the literature (Browning and Chiappori (1998), Browning et al. (2014), Bourguignon et al. (2009)) and rely on the distribution proportionality property. This property re-

Table A15: Testing Pareto Efficiency

	Sample		
	All Households	Nuclear Only	Extended Only
	(1)	(2)	(3)
Test of equality of ratios between:			
1) Men's Clothing and Food Shares			
Wald statistic	0.059	0.004	0.286
p-value	0.809	0.947	0.593
2) Men's Clothing, Women's Clothing, and Food Budget Shares			
Wald statistic	0.771	0.044	0.558
p-value	0.680	0.978	0.757
3) Men's Clothing , Women's Clothing, Children's Clothing, and Food Budget Shares			
Wald statistic	0.785	1.620	1.119
p-value	0.853	0.655	0.773

Malawi Integrated Household Survey and Integrated Household Panel Survey. Tests for proportionality restriction of the effects of distribution factors (matrilineal village and average education differences across adult men and women) across outcomes (Browning and Chiappori, 1998). The underlying regression models include the same household level controls used in the main estimation results. Only households with one married woman and one married man are included in Column 2. Only households with more than one woman or more than one man are included in Column 3.

quires the ratio of the impact of two distribution factors on demand to be proportional across goods.

I estimate the following Engel curves:

$$W_s^k = \alpha_0^k + \beta_1^k d_s^1 + \beta_2^k d_s^2 + \mathbf{X_s}'\gamma_\mathbf{s}^\mathbf{k} + \epsilon_s^k \tag{A.5}$$

where W_s^k is either the household's food budget share ($k =$ food), or clothing budget shares for men, women, and children ($k =$ men's, women's, or children's clothing) in a household of type s. The distribution factors are given by d_s^1 and d_s^2. I include a vector of household characteristics (the same presented in Table A18).

Pareto efficiency holds when $\frac{\beta_1^k}{\beta_2^k} = \frac{\beta_1^j}{\beta_2^j}$ for $j \neq k$. I follow Brown et al. (2018) and conduct the tests separately for all households, nuclear households, and extended family households. I use a non-linear Wald test for the equality of the ratios, and consistently fail to reject them. The results are presented in Table A15. The obvious caveat to these tests is that they are dependant on the validity of the distribution factors.

A.7 Ratio Restriction Examples

The following tables illustrate examples of the ratio restrictions for households with two or fewer children. Recall that Ratio Restriction 1 takes the following form for non-foster children: $\frac{\eta^a_{s_{a0}}}{\eta^a_{s_{a+1,0}}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{s_{a+1,b}}}$. The resource share values below are consistent with this restriction.

Household	# Non-Foster	# Foster	$\eta^a_{s_{ab}}$	Ratio Restriction 1
A	1	0	20	
B	2	0	15	
C	1	1	16	
D	2	1	$\eta^a_{21} = 12$	$\frac{16}{\eta^a_{21}} = \frac{20}{15} \rightarrow \frac{16}{12} = \frac{20}{15}$

Household A has one non-foster child and zero foster children. That child consumes 20 percent of the budget. Household B has two non-foster children and *each* non-foster child consumes 15 percent of the budget. So adding a non-foster children decreased the non-foster child resource shares by 25 percent, when there are no foster children present. According to Ratio Restriction 1, this 25 percent decline must be independent of the number of foster children present in the household. That is, when their is one foster child present, we still see the 25 percent decline (non-foster child resource shares decrease from 16 to 12.)

Ratio Restriction 2 requires that $\frac{\eta^a_{s_{10}}}{\eta^b_{s_{01}}} = \frac{\eta^a_{s_{11}}}{\eta^b_{s_{11}}}$. To better understand this assumption, consider the following example:

Household	# Non-Foster	# Foster	$\eta^a_{s_{ab}}$	$\eta^b_{s_{ab}}$	Ratio Restriction 2
A	1	0	20	0	
B	0	1	0	20	
C	1	1	16	$\eta^b_{11} = 16$	$\frac{20}{20} = \frac{\eta^a_{10}}{\eta^b_{01}} = \frac{\eta^a_{11}}{\eta^b_{11}} = \frac{16}{16}$

Here, Households A and B are one-child-type, whereas Household C is a composite household. Ratio Restriction 2 requires that foster and non-foster child resource shares in Household C, $\eta^a_{11}$and η^b_{11}, are proportional to foster and non-foster child resource shares in Households A and B. In particular, if $\eta^a_{10} = 20$, and $\eta^b_{01} = 20$, then $\frac{\eta^a_{11}}{\eta^b_{11}} = \frac{20}{20}$. Importantly, this restriction applies to only a single composite household type.

A.8 Fully Specified Model

In this section, I follow Dunbar et al. (2013) and write a fully specified household model that is consistent with the restrictions contained in Section **??**.

Let y be household expenditure, and $\tilde{\mathbf{p}}$ be the price vector of all goods aside from the private assignable goods given by p_t. While more general formulations are possible, I start with assuming individuals have sub-utility over goods given by the Price Independent Generalized Logarithmic (Piglog) functional form (Deaton and Muellbauer, 1980).

$$\ln V_t(\mathbf{p}, y) = \ln[\ln(\frac{y}{G^t(p_t, \tilde{\mathbf{p}})})] + p_t e^{-a' \ln \tilde{\mathbf{p}}} \tag{A.6}$$

where G^t is some function that is non-zero, differentiable, and homogeneous of degree one, and some constant vector a with elements a_k summing to one. Each member of the same type is assumed to have the same utility function. This assumption can be relaxed with a data set that has goods that are assignable at a more detailed level.

The household weights individual utilities using the following Bergson-Samuelson social welfare function:

$$\tilde{U}_s(U_f, U_m, U_a, U_b, \mathbf{p}/y) = \sum_{t \epsilon \{m,f,a,b\}} \omega_t(\mathbf{p})[U_t + \rho_t(\mathbf{p})] \tag{A.7}$$

where $\omega_t(\mathbf{p})$ are the Pareto weight functions and $\rho_t(\mathbf{p})$ are the externality functions. Individuals are allowed to receive utility from another person's utility, but not from another person's consumption of a specific good. This can be considered a form of restricted altruism.

The household's problem is to maximize the social welfare function subject to a budget constraint, and a consumption technology constraint.

$$\begin{aligned} \max_{x_m, x_f, x_a, x_b, z_s} \quad & \omega(\mathbf{p}) + \sum_{t \epsilon \{m,f,a,b\}} \omega_t(\mathbf{p}) U_t \\ & \text{s.t} \quad y = \mathbf{z_s'} \mathbf{p} \ \text{ and} \\ & z_s^k = A_s^k (x_m^k + x_f^k + \sigma_a x_a^k + \sigma_b x_b^k) \ \text{ for each good } \ k \end{aligned}$$

where the household type is given by s, or the number of foster and non-foster children present in the household, and $\omega(\mathbf{p}) = \sum_{t \epsilon \{m,f,a,b\}} \omega_t(\mathbf{p}) \rho_t(\mathbf{p})$. Matrix $\mathbf{A_s}$ is the consumption technology function. It is a $k \times k$ diagonal matrix and determines the relative publicness or privateness of good k. If good k is private, then the k,k'th element is equal to one, and what the household purchases is exactly equal to individual consumption.

By Pareto efficiency, the household maximisation can be decomposed into two step process; In the first stage, resource shares are optimally allocated. In the second stage, each individual maximizes their individual utility subject to the budget constraint $A_s^k p^k x_t^k = \eta_s^t y$. Resource shares are defined as $\eta_s^t = \mathbf{x^t} \mathbf{A_s} \mathbf{p}/y = \sum_k A_s^k p^k x_t^k / y$ evaluated at the optimized level of expenditures x_t. The optimal utility level is given by the individual's indirect utility function V^t evaluated at Lindahl prices, $V_t(\mathbf{A_s'p}, \eta_s^t, y)$.

Using the functional form assumptions regarding individual indirect utility functions, the household problem can be rewritten:

$$\max_{\eta_s^m,\eta_s^f,\eta_s^a,\eta_s^b} \quad \omega(\mathbf{p}) + \sum_{t\epsilon\{m,f,a,b\}} \tilde{\omega}_s^t(\mathbf{p}) \ln(\frac{\eta_s^t y}{G^t(\mathbf{A_s'p})})$$
$$\text{s.t} \quad \eta_s^m + \eta_s^f + \sigma_a \eta_s^a + \sigma_b \eta_s^b = 1 \tag{A.8}$$

where $\tilde{\omega}(\mathbf{p}) = \omega_t \exp(\mathbf{A_t} p_t e^{-a'(\ln \tilde{\mathbf{p}} + \ln \tilde{\mathbf{A}}_s)})$

The first order conditions from this maximisation problem are as follows:

$$\frac{\tilde{\omega}_s^m(\mathbf{p})}{\eta_s^m} = \frac{\tilde{\omega}_s^f(\mathbf{p})}{\eta_s^f} = \frac{\tilde{\omega}_s^a(\mathbf{p})}{\sigma_a \eta_s^a} = \frac{\tilde{\omega}_s^b(\mathbf{p})}{\sigma_b \eta_s^b}, \quad \text{and} \quad \sum_{t\epsilon\{m,f,a,b\}} \sigma_t \eta_s^t = 1 \tag{A.9}$$

Solving for person specific resource shares gives the following equations:

$$\eta_s^t(\mathbf{p}) = \frac{\tilde{\omega}_s^t(\mathbf{p})}{\tilde{\omega}_s^m + \tilde{\omega}_s^f + \tilde{\omega}_s^a + \tilde{\omega}_s^b} \text{for } t \ \epsilon \ \{m, f\} \tag{A.10}$$

$$\eta_s^t(\mathbf{p}) = \frac{\tilde{\omega}_s^t(\mathbf{p})/\sigma_t}{\tilde{\omega}_s^m + \tilde{\omega}_s^f + \tilde{\omega}_s^a + \tilde{\omega}_s^b} \text{for } t \ \epsilon \ \{a, b\} \tag{A.11}$$

With each person now allocated their share of household resources, each person can then maximize there own utility, subject to their own personal budget constraint. In particular, individuals choose x_t to maximize $U_t(\mathbf{x_t})$ subject to $\eta_s^t y = \sum_k A_s^k p_k x_t^k$. Individual demand functions are derived using Roy's Identify on the indirect utility functions given in Equation (A.23), where individual income is used $\eta_s^t y$ and individuals face the Lindahl price vector $\mathbf{A_s p}$.

$$h_t^k(\eta_s^t y, \mathbf{A_s p}) = \frac{\eta_s^t y}{G^t} \frac{\partial G^t}{\partial \mathbf{A_s} p^k} - \frac{\partial(\mathbf{A} p^k e^{-a' \ln \tilde{\mathbf{p}}})}{\partial \mathbf{A} p^k} [\ln \eta_s^t y - \ln G^t] \eta_s^t y \tag{A.12}$$

for any good k for person of type t. This can be written more concisely:

$$h_t^k(\eta_s^t y, \mathbf{A_s'p}) = \tilde{\delta}_t^k(\mathbf{A_s'p}) \eta_s^t y - \psi_t^k(\mathbf{A_s'p}) \eta_s^t y \ln(\eta_s^t y) \tag{A.13}$$

Budget shares $h_t^k(\eta_s^t y, \mathbf{A_s'p})/y$ are required to be between zero and one. Furthermore, the adding up constraint requires that budget shares sum to one:[7]

$$\sum_k \frac{h_t^k(\eta_s^t y, \mathbf{A_s'p})}{y} = 1 \tag{A.14}$$

Using the individual demand functions, household demand for good k is written in general terms as follows accounting for the consumption technology function:

$$z_s^k = \mathbf{A_s} \sum_{t \epsilon \{m,f,a,b\}} h_t^k(\mathbf{A_s'p}, \eta_s^t(\mathbf{p})y) \tag{A.15}$$

Dividing the individual demand functions by household expenditure produces the budget share equations:

$$\frac{h_t^k(\eta_s^t y, \mathbf{A_s'p})}{y} = \tilde{\delta}_t^k(\mathbf{A_s'p})\eta_s^t - \psi_t^k(\mathbf{A_s'p})\eta_s^t \ln(\eta_s^t y) \tag{A.16}$$

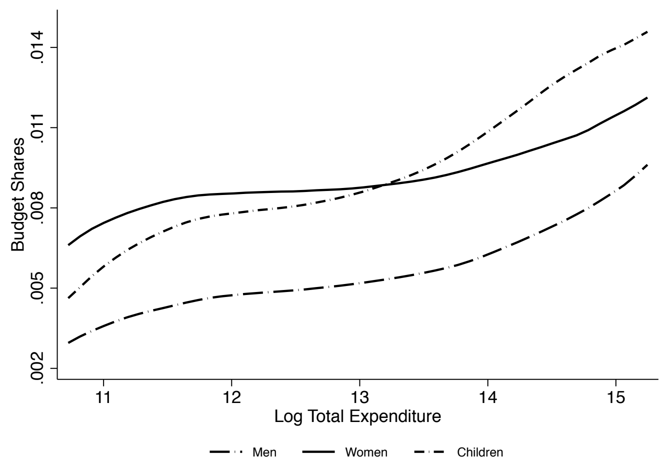
The analysis in this paper uses Engel curves for private goods, which simplifies the above equations even further. First, Engel curves demonstrate how budget shares vary with income holding prices constant. Thus prices can be dropped from the above equation. Secondly, the consumption technology drops out for private goods, as the element in the A matrix takes a value of 1 for private goods. The Engel curves are then written as follows:

$$W_s^t(y) = \frac{h_s^t(y)}{y} = \eta_s^t \delta_s^t + \eta_s^t \beta_s^t (\ln y + \ln \eta_s^t) \tag{A.17}$$

[7] In the estimation of the model, the adding up constraint is ignored as I am not estimating a full demand system.

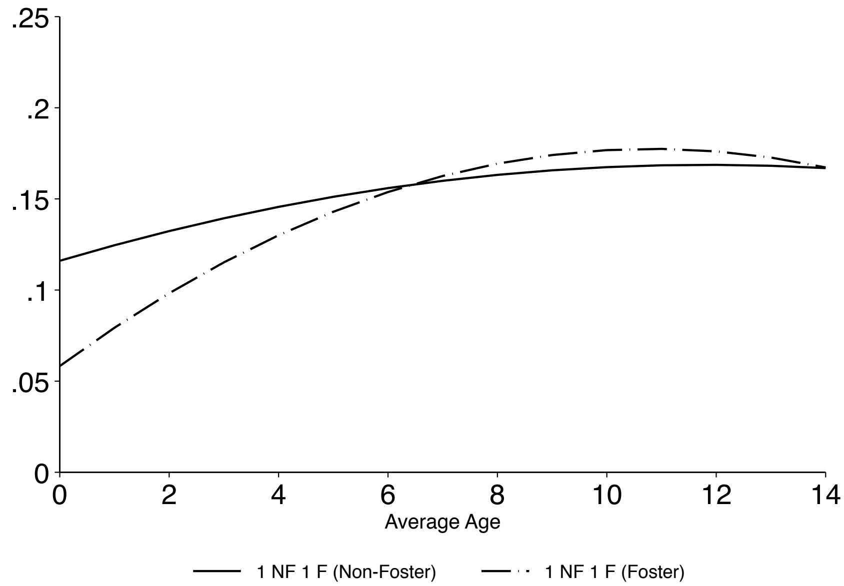
A.9 Additional Figures

Figure A1: Clothing Engel Curves



Notes: The Figure displays non-parametric clothing Engel curves for men, women, and children.

Figure A2: Predicted Resource Shares by Child Age



Notes: The Figure plots predicted resource shares in households with one child of each type ("1 NF 1 F"). The predictions are made for a reference household, whish is defined as having all covariates at their median value, except for age, which is varied from zero to fourteen. Solid lines are non-foster child resource shares. Dashed lines are foster child resource shares.

A.10 Additional Tables

Table A16: Household Structure

		# Foster				
		0	1	2	3	4
	0	0	872	417	182	78
	1	3,948	501	135	40	0
# Non-Foster	2	4,188	409	95	0	0
	3	3,744	261	0	0	0
	4	2,333	0	0	0	0

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes households with 1-4 men and women, and 1-4 children.

Table A17: Slope Preference Parameters

Preference Restriction:	SAT	SAT	SAP	SAP+SAT
One-Child-Type and Composite Similarity:	Yes	Yes	No	Yes
Ratio Restrictions:	No	Yes	Yes	Yes
	(1)	(2)	(3)	(4)
SAT				
β^a	0.0167***	0.0170***		
	(0.00354)	(0.00328)		
β^b	0.00799***	0.00722**		
	(0.00306)	(0.00290)		
$\beta^f = \beta^m$	0.00937***	0.00936***		
	(0.000747)	(0.000690)		
SAP				
β_{one}			0.0132***	
			(0.00126)	
$\beta_{s_{nonfoster}}$			0.000461	
			(0.000326)	
$\beta_{s_{foster}}$			-0.00107**	
			(0.000537)	
$\beta_{s_{men}}$			-0.00130***	
			(0.000499)	
$\beta_{s_{women}}$			-0.00108**	
			(0.000521)	
SAT + SAP				
β				0.0106***
				(0.00049)
Sample Size	17,203	17,203	17,203	17,203
Log Likelihood	150,455	150,467	150,487	150,453

Notes: Malawi Integrated Household Survey, and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. The table presents parameter estimates for the slope preference parameter β_s^t for the four sets of identification assumptions displayed in Table **??**. "SAT" is the similar across household types assumption. "SAP" is the similar across person types assumption. In columns (1) and (2), I allow slope preferences to differ across foster children, non-foster children, and adults, but they are assumed to be identical across household sizes. In column (3), β_s can differ across household sizes (i.e., the number of each person type), but not person types. In column (4), the slope preference parameter is assumed to be identical across household sizes and person types. * p<0.1, ** p<0.05, *** p<0.01

Table A18: *Determinants of Resource Shares: Preference Factors*

	Non-Foster Children (1) NLSUR	Foster Children (2) NLSUR
North	-0.00310	0.0101
	(0.0134)	(0.0148)
Central	-0.00212	-0.00227
	(0.00917)	(0.0118)
Year=2010	-0.0258**	-0.0172
	(0.0101)	(0.0111)
Year=2013	-0.0202	0.00228
	(0.0137)	(0.0146)
Average Age non-Foster	1.435**	-0.227
	(0.586)	(0.771)
Average Age non-Foster^2	-0.0811*	0.0186
	(0.0418)	(0.0546)
Average Age Foster	-0.545	2.428
	(1.888)	(1.609)
Average Age Foster^2	0.0435	-0.120
	(0.107)	(0.102)
Proportion of Fostered Orphaned	0.0265	-0.0190
	(0.0259)	(0.0289)
Fraction Female non-Foster	-0.0146	-0.00524
	(0.0112)	(0.0174)
Fraction Female Foster	0.00794	-0.0295
	(0.0268)	(0.0333)
Average Age Women	0.462*	0.0613
	(0.270)	(0.288)
Average Age Women^2	-0.00786**	-0.00130
	(0.00354)	(0.00356)
(Average Age Men - Average Age Women)	-0.0855	-0.0291
	(0.0584)	(0.0566)
(Average Age Men - Average Age Women)^2	0.00260	0.00207
	(0.00179)	(0.00191)
Average Education Men	-0.00638	0.00178
	(0.00792)	(0.0107)
Average Education Women	-0.00539	-0.00644
	(0.00820)	(0.0104)
Rural	0.00277	-0.000770
	(0.0116)	(0.0123)
Share of Adult Women Age 15-18	0.0336	0.0175
	(0.0331)	(0.0337)
Share of Adult Men Age 15-18	0.000851	0.0185
	(0.0242)	(0.0253)
Matrilineal Village	0.00777	0.0180
	(0.00948)	(0.0118)
Sample Size	17,203	
Log Likelihood	150,467	

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. The education and age variables are demeaned. South Malawi is the omitted region. Coefficients on the household composition indicators are omitted for conciseness. * p<0.1, ** p<0.05, *** p<0.01

Table A19: Determinants of Resource Shares: Household Type Indicators

	Non-Foster Children (1) NLSUR	Foster Children (2) NLSUR
1 non-Foster 0 Foster	0.188***	
	(0.0400)	
2 non-Foster 0 Foster	0.251***	
	(0.0484)	
3 non-Foster 0 Foster	0.285***	
	(0.0548)	
4 non-Foster 0 Foster	0.316***	
	(0.0622)	
0 non-Foster 1 Foster		0.213***
		(0.0455)
1 non-Foster 1 Foster	0.151***	0.241***
	(0.0313)	(0.0530)
2 non-Foster 1 Foster	0.201***	0.154***
	(0.0381)	(0.0342)
3 non-Foster 1 Foster	0.228***	0.138***
	(0.0435)	(0.0335)
0 non-Foster 2 Foster		0.301***
		(0.0637)
1 non-Foster 2 Foster	0.148***	0.241***
	(0.0381)	(0.0530)
2 non-Foster 2 Foster	0.197	0.218***
	(0.0479)	(0.0501)
0 non-Foster 3 Foster		0.363***
		(0.0839)
1 non-Foster 3 Foster	0.129***	0.291***
	(0.0482)	(0.0703)
0 non-Foster 4 Foster		0.428***
		(0.104)
No. Men	-0.00270	-0.00341
	(0.00697)	(0.00741)
No. Women	-0.00392	-0.00252
	(0.00955)	(0.00899)
Sample Size	17,203	
Log Likelihood	150,467	

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. The education and age variables are demeaned. Coefficients on the covariates (age, education, etc.) are omitted for conciseness. The parameter estimates are not per child, but rather the total resources allocated to foster or non-foster children. * $p<0.1$, ** $p<0.05$, *** $p<0.01$

Table A20: Determinants of Resource Shares: Alternative Household Types

	Non-Foster Children			Foster Children		
	Main Results (1a)	Nuclear Households (2a)	No Polygamous Households (3a)	Main Results (1b)	Nuclear Households (2b)	No Polygamous Households (3b)
Household Type Indicators						
2 Non-Foster 0 Foster	0.251***	0.246***	0.250***			
	(0.0484)	(0.0521)	(0.0497)			
1 Non-Foster 1 Foster	0.151***	0.161***	0.145***	0.241***	0.324***	0.226***
	(0.0313)	(0.0358)	(0.0321)	(0.0530)	(0.0904)	(0.0551)
0 Non-Foster 2 Foster				0.301***	0.353***	0.286***
				(0.0637)	(0.0984)	(0.0660)
3 Non-Foster 0 Foster	0.285***	0.285***	0.288***			
	(0.0548)	(0.0607)	(0.0566)			
2 Non-Foster 1 Foster	0.201***	0.225***	0.197***	0.154***	0.155***	0.145***
	(0.0381)	(0.0465)	(0.0394)	(0.0342)	(0.0550)	(0.0343)
1 Non-Foster 2 Foster	0.148***	0.115***	0.140***	0.241***	0.324***	0.226***
	(0.0381)	(0.0390)	(0.0393)	(0.0530)	(0.0904)	(0.0551)
0 Non-Foster 3 Foster				0.363***	0.426***	0.344***
				(0.0839)	(0.132)	(0.0873)
Covariates						
Average Age non-Foster	1.435**	1.936***	1.258**	-0.227	-0.331	-0.263
	(0.586)	(0.712)	(0.607)	(0.771)	(1.187)	(0.753)
Average Age non-Foster^2	-0.0811*	-0.134**	-0.0705	0.0186	0.0424	0.0199
	(0.0418)	(0.0533)	(0.0433)	(0.0546)	(0.0852)	(0.0537)
Average Age Foster	-0.545	-1.274	-0.879	2.428	0.952	2.251
	(1.888)	(2.029)	(2.086)	(1.609)	(2.205)	(1.508)
Average Age Foster^2	0.0435	0.0710	0.0621	-0.120	-0.0142	-0.114
	(0.107)	(0.115)	(0.117)	(0.102)	(0.144)	(0.0954)
Proportion Non-Foster Female	-0.0146	-0.0156	-0.0136	-0.00524	-0.0314	-0.00711
	(0.0112)	(0.0129)	(0.0118)	(0.0174)	(0.0309)	(0.0171)
Proportion Foster Female	0.00794	-0.0159	0.00702	-0.0295	0.00160	-0.0271
	(0.0268)	(0.0331)	(0.0273)	(0.0333)	(0.0429)	(0.0329)
Rural	0.00277	-0.0123	0.00278	-0.000770	0.00348	-0.00106
	0.00277	-0.0123	0.00278	-0.000770	0.00348	-0.00106
Matrilineal Village	0.00777	-0.000473	0.00926	0.0180	0.00425	0.0171
	(0.00948)	(0.0116)	(0.01000)	(0.0118)	(0.0234)	(0.0113)
Proportion of Fostered Orphaned	0.0265	0.0203	0.0324	-0.0190	-0.0192	-0.0233
	(0.0259)	(0.0320)	(0.0265)	(0.0289)	(0.0455)	(0.0288)
Sample Size	17,203	9,609	15,816	17,203	9,609	15,816
Log Likelihood	150,467	83,283	138,075	150,467	83,283	138,075

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Estimates for certain household types and preferences factors are omitted for conciseness. The parameters on the household type indicators are not per child, but the total allocation to all foster or non-foster children within the household. * p<0.1, ** p<0.05, *** p<0.01

Table A21: Determinants of Resource Shares: Accounting for Wealth

	Non-Foster Children		Foster Children	
	Main Results	Wealth in Resource Function	Main Results	Wealth in Resource Function
	(1a)	(2a)	(1b)	(2b)
Household Type Indicators				
2 Non-Foster 0 Foster	0.251***	0.249***		
	(0.0484)	(0.0459)		
1 Non-Foster 1 Foster	0.151***	0.147***	0.241***	0.252***
	(0.0313)	(0.0293)	(0.0530)	(0.0502)
0 Non-Foster 2 Foster			0.301***	0.325***
			(0.0637)	(0.0611)
3 Non-Foster 0 Foster	0.285***	0.280***		
	(0.0548)	(0.0514)		
2 Non-Foster 1 Foster	0.201***	0.193***	0.154***	0.168***
	(0.0381)	(0.0350)	(0.0342)	(0.0337)
1 Non-Foster 2 Foster	0.148***	0.150***	0.241***	0.252***
	(0.0381)	(0.0367)	(0.0530)	(0.0502)
0 Non-Foster 3 Foster			0.363***	0.399***
			(0.0839)	(0.0806)
			(0.0833)	(0.0799)
Covariates				
Log Value of Household Assets		-0.000152		-0.000152
		(0.000966)		(0.000966)
Average Age non-Foster	1.435**	1.821***	-0.227	-0.293
	(0.586)	(0.601)	(0.771)	(0.787)
Average Age non-Foster^2	-0.0811*	-0.104**	0.0186	0.0212
	(0.0418)	(0.0422)	(0.0546)	(0.0558)
Average Age Foster	-0.545	-0.537	2.428	2.761*
	(1.888)	(1.997)	(1.609)	(1.415)
Average Age Foster^2	0.0435	0.0442	-0.120	-0.137
	(0.107)	(0.112)	(0.102)	(0.0947)
Proportion Non-Foster Female	-0.0146	-0.0140	-0.00524	-0.00870
	(0.0112)	(0.0108)	(0.0174)	(0.0183)
Proportion Foster Female	0.00794	0.00882	-0.0295	-0.0337
	(0.0268)	(0.0258)	(0.0333)	(0.0345)
Rural	0.00277	0.00193	-0.000770	-0.000117
	0.00277	0.00193	-0.000770	-0.000117
Matrilineal Village	0.00777	0.00725	0.0180	0.0200
	(0.00948)	(0.00926)	(0.0118)	(0.0127)
Proportion of Fostered Orphaned	0.0265	0.0219	-0.0190	-0.0172
	(0.0259)	(0.0252)	(0.0289)	(0.0287)
Sample Size	17,203	17,203	17,203	17,203
Log Likelihood	150,467	150,509	150,467	150,509

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all households with 1-4 men and women, and 1-4 children. Robust standard errors in parentheses. Age variables are divided by 100 to ease computation. Estimates for certain household types and preferences factors are omitted for conciseness. The parameters on the household type indicators are not per child, but the total allocation to all foster or non-foster children within the household. * p<0.1, ** p<0.05, *** p<0.01

Table A22: Descriptive Statistics: Education and Child Labour

	Mean	Std. Dev.	Min	Max	Sample Size
Foster Status					
Both Parents Present	0.598	0.490	0	1	35,198
Father Present Mother Absent and Alive	0.016	0.125	0	1	35,198
Father Present Maternal Orphan	0.008	0.089	0	1	35,198
Mother Present Father Absent and Alive	0.143	0.350	0	1	35,198
Mother Present Paternal Orphan	0.056	0.230	0	1	35,198
Both Absent and Alive	0.113	0.317	0	1	35,198
Double Orphan	0.023	0.149	0	1	35,198
Both Absent Paternal Orphan	0.023	0.150	0	1	35,198
Both Absent Maternal Orphan	0.020	0.140	0	1	35,198
Individual and Household Characteristics					
Enrolled in School	0.886	0.317	0	1	35,198
Hours Worked in Chores Past Week	1.604	5.115	0	96	35,198
Hours Worked (Excluding Chores) Past Week	2.369	4.457	0	49	35,198
Log Expenditure per Capita	11.792	0.662	9.308	15	35,198
Log Remmitances Per Capita	0.779	6.130	-4.554	15	35,198
North	0.206	0.404	0	1	35,198
Central	0.355	0.478	0	1	35,198
South	0.440	0.496	0	1	35,198
Year = 2010	0.428	0.495	0	1	35,198
Year = 2013	0.151	0.358	0	1	35,198
Year = 2016	0.421	0.494	0	1	35,198
Male Sibling Age 0-6	0.264	0.581	0	5	35,198
Female Siblings Age 0-6	0.267	0.586	0	4	35,198
Male Siblings Age 7-14	0.277	0.603	0	6	35,198
Female Siblings Age 7-14	0.273	0.589	0	6	35,198
Men	1.302	0.928	0	9	35,198
Women	1.449	0.773	0	7	35,198
Age	9.767	2.580	6	14	35,198
Female	0.507	0.500	0	1	35,198
Urban	0.831	0.374	0	1	35,198
Age Household Head	44.359	12.994	13	104	35,198
Female Household Head	0.264	0.441	0	1	35,198
Education of Household Head	1.086	0.713	0	3	35,198

Notes: Malawi Integrated Household Survey and Integrated Household Panel Survey. The sample includes all children age 6 to 14.

A.11 Identification Theorems

What follows are extended versions of the identification theorems in Dunbar et al. (2013). Theorem 1 demonstrates how resource shares can be identified using the SAT restriction, while Theorem 2 does the same using the SAP restriction. Parts of both theorems and their respective proofs are similar to what is found in Dunbar et al. (2013). I discuss where and why I differ throughout the Theorems.

Let $h_t^k(\mathbf{p}, y)$ be the Marshallian demand function for good k and let the consumption utility function of person t be defined as $U_t(\mathbf{x_t})$. Individual t chooses $\mathbf{x_t}$ to maximize $U_t(\mathbf{x_t})$ under the budget constraint $\mathbf{p}'\mathbf{x_t} = y$ with $\mathbf{x_t} = \mathbf{h_t}(\mathbf{p}, y)$ for all goods k. Define the indirect utility function $V_t(\mathbf{p}, y) = U_t(\mathbf{h_t}(\mathbf{p}, y))$ where $\mathbf{h_t}(\mathbf{p}, y)$ is the vector of demand functions for all goods k.

The household solves the following maximisation problem where each individual person type has their own utility function:[8]

$$\begin{aligned} \max_{x_m, x_f, x_a, x_b} \tilde{U}_{s_{ab}}[U_m(\mathbf{x_m}), U_f(\mathbf{x_f}), U_a(\mathbf{x_a}), U_b(\mathbf{x_b}), \mathbf{p}/y] \text{ such that} \\ \mathbf{z_{s_{ab}}} = \mathbf{A_{s_{ab}}}[\mathbf{x_m} + \mathbf{x_f} + \sigma_a \mathbf{x_a} + \sigma_b \mathbf{x_b}] \text{ and} \\ y = \mathbf{z}'\mathbf{p} \end{aligned} \tag{A.18}$$

The household demand functions are given by $H^k_{s_{ab}}(\mathbf{p}, y)$. Let $\mathbf{A^k_{s_{ab}}}$ be the row vector given by the k'th row of the linear technology function $\mathbf{A_{s_{ab}}}$. Each individual faces the shadow budget constraint defined by the Lindahl price vector $\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}$ and individual income $\eta^t_{s_{ab}} y$. Then household demand can be written as follows:

$$z^k_{s_{ab}} = H^k_{s_{ab}}(\mathbf{p}, y) = \mathbf{A^k_{s_{ab}}}\Big[\sum_{t \in \{m,f,a,b\}} \sigma_t h^k_t(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}, \eta^t_{s_{ab}} y)\Big] \tag{A.19}$$

where $\eta^t_{s_{ab}}$ are the resource shares of person t in a household with σ_a foster children and σ_b non-foster children. Resource shares by construction must sum to one.

$$\eta^m_{s_{ab}} + \eta^f_{s_{ab}} + \sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}} = 1 \tag{A.20}$$

ASSUMPTION A1: Equations (A.18), (A.19), and (A.20) hold with resource shares $\eta^t_{s_{ab}}$ that do not depend on y.

Resource shares being independent of household expenditure is the key identifying assump-

[8] For simplicity, I have assumed there are one man and one woman in each household.

tion. Resource shares can still depend on other variables correlated with household expenditure such as the individual wages for men and women.

DEFINITION: A good k is a *private* good if, for any household size s_{ab}, the matrix $\mathbf{A}_{\mathbf{s_{ab}}}$, has a one in position k, k and has all other elements in row k and column k equal to zero.

DEFINITION: A good k is an *assignable* good if it only appears in one of the utility functions U_m, U_f, U_a, and U_b.

Men's and women's clothing expenditures are examples of private assignable goods. These goods are central to identification in Dunbar et al. (2013) and they are here as well. What makes private assignable goods unique and especially useful for identification is that by definition, the quantities that the household purchases are equivalent to what individuals in the household consume. In other words, there are no economies of scale or sharing for these goods making household-level consumption in some sense equivalent to individual-level consumption. However, because I lack a private assignable good for foster and non-foster children, I must make use of partially assignable goods.

DEFINITION: A good k is a *partially assignable* good if it only appears in two of the utility functions U_m, U_f, U_a, and U_b.

An example of a partially assignable good is children's clothing, which are partially assignable to foster and non-foster children. Specifically, children's clothing only appears in the utility functions for foster and non-foster children, U_a and U_b. In other contexts, children's clothing expenditures can be classified as partially assignable to boys and girls, or potentially to young and old children. Other examples of partially assignable goods commonly found in household survey data include alcohol and tobacco, which are assignable to adults, but only partially assignable to adult men and women.

The distinction between assignable and partially assignable goods is in some ways determined by the question the researcher is interested in answering. For example, Dunbar et al. (2013) are interested in estimating intrahousehold inequality between men, women, and children within the household, and are therefore less interested in understanding inequality *among* children within the household, as I am in this context. They assume all children have the same utility function, U_c, or that $U_a = U_b$. As a result, children's clothing expenditures are assignable, as they only appear in U_c. In my context, where I allow foster and non-foster children to have different utility functions and ultimately different resource shares, children's clothing expenditures now appear in both U_a and U_b and are therefore no longer assignable.

ASSUMPTION A2: Assume that the demand functions include a private assignable good for men and women, denoted as goods m and f. Assume that the demand functions include a private partially assignable good for foster and non-foster children, denoted as good c.

The household demand functions for the private assignable goods for men and women can be written as follows:

$$z^k_{s_{ab}} = H^k_{s_{ab}} = h^k(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}, \eta^k_{s_{ab}}(\mathbf{p})y) \text{ for } k \epsilon \{m, f\} \tag{A.21}$$

For the foster and non-foster children, household demand functions for the private partially assignable good can be written as follows:

$$z^c_{s_{ab}} = H^c_{s_{ab}} = \sigma_a h^a(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}, \eta^a_{s_{ab}}(\mathbf{p})y) + \sigma_b h^b(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}, \eta^b_{s_{ab}}(\mathbf{p})y) \tag{A.22}$$

In practice, I take the household demand functions for foster child clothing, and non-foster child clothing, and sum them together. Taking this action is possible since the goods are private. In the empirical application, this means that I assume clothing is not shared across child types.

Define p_m and p_f to be the prices of the private assignable goods and define p_c to be the price of the private partially assignable good. Define $\bar{\mathbf{p}}$ to be the vector of prices for all private goods excluding p_m, p_f, and p_c. Assume $\bar{\mathbf{p}}$ is nonempty. Let $\tilde{\mathbf{p}}$ be the vector of all non-private goods.

ASSUMPTION A3: Each person $t \epsilon \{m, f, a, b\}$ has the following indirect utility function:[9]

$$V_t(\mathbf{p}, y) = \psi_t\Big[u_t\Big(\frac{y}{G^t(\tilde{\mathbf{p}})}, \frac{\bar{\mathbf{p}}}{p_t}\Big), \tilde{\mathbf{p}}\Big] \tag{A.23}$$

where G^t is some function that is nonzero, differentiable, and homogeneous of degree one, ψ_t and u_t are strictly positive, differentiable, and strictly monotonically increasing in their first arguments, and differentiable and homogenous of degree zero in their remaining elements.[10]

By Roy's identity, the demand functions for the private assignable goods $k \epsilon \{m, f, a, b\}$ can

[9] As discussed in DLP, the indirect utility function only has to take this form for low levels of expenditure. For simplicity, I assume the indirect utility function is the same across all expenditure levels.

[10] Assumption A3 is a modified version of Assumption B3 in Dunbar et al. (2013).

be written as follows:

$$h^k(y, \mathbf{p}) = \frac{\partial u_k\Big(\frac{y}{G^k(\tilde{\mathbf{p}})}, \frac{\bar{\mathbf{p}}}{p_k}\Big)'}{\partial(\bar{\mathbf{p}}/p_k)} \frac{\bar{\mathbf{p}}}{p_k^2} \frac{G^k(\tilde{\mathbf{p}})}{u_k'\Big(\frac{y}{G^k(\tilde{\mathbf{p}})}, \frac{\bar{\mathbf{p}}}{p_k}\Big)} = \tilde{f}_k\Big(\frac{y}{G^k(\tilde{\mathbf{p}})}, p_k, \bar{\mathbf{p}}\Big)y$$

Since p_k and $\bar{\mathbf{p}}$ do not change when replaced by $\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}$, substituting the above equation into Equation (A.21) gives the household demand functions for the assignable goods:

$$H^k_{s_{ab}}(y, \mathbf{p}) = \tilde{f}_k\Big(\frac{\eta^k_{s_{ab}}(\mathbf{p})y}{G^k(\tilde{\mathbf{A}}'_{s_{ab}}\tilde{\mathbf{p}})}, p_k, \bar{\mathbf{p}}\Big)\eta^k_{s_{ab}}(\mathbf{p})y$$

The Engel curve by definition holds price constant, and can then be written as:

$$H^k_{s_{ab}}(y) = \tilde{f}_k\Big(\frac{\eta^k_{s_{ab}}y}{G^k_{s_{ab}}}\Big)\eta^k_{s_{ab}}y \tag{A.24}$$

However, because there are no private assignable goods for foster and non-foster children, I write the Engel curve for the private partially assignable good for children in place of $H^a_{s_{ab}}$ and $H^b_{s_{ab}}$ as follows:

$$H^c_{s_{ab}}(y) = \tilde{f}_a\Big(\frac{\eta^a_{s_{ab}}y}{G^a_{s_{ab}}}\Big)\sigma_a\eta^a_{s_{ab}}y + \tilde{f}_b\Big(\frac{\eta^b_{s_{ab}}y}{G^b_{s_{ab}}}\Big)\sigma_b\eta^b_{s_{ab}}y \tag{A.25}$$

Define the matrix Ω' by

$$\Omega' = \begin{bmatrix}
\frac{\eta^m_{10}}{\eta^m_{20}} & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\eta^f_{10}}{\eta^f_{20}} & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\eta^m_{01}}{\eta^m_{02}} & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\eta^f_{01}}{\eta^f_{02}} & -1 \\
0 & -1 & 0 & 0 & 0 & \frac{\eta^m_{10}}{\eta^m_{01}} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{\eta^f_{10}}{\eta^f_{01}} \\
\frac{\eta^m_{10}}{\eta^m_{20}} - \frac{\eta^a_{10}}{\eta^a_{20}} & 0 & \frac{\eta^f_{10}}{\eta^f_{20}} - \frac{\eta^a_{10}}{\eta^a_{20}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\eta^m_{01}}{\eta^m_{02}} - \frac{\eta^b_{01}}{\eta^b_{02}} & 0 & \frac{\eta^f_{01}}{\eta^f_{02}} - \frac{\eta^b_{01}}{\eta^b_{02}} & 0
\end{bmatrix}$$

ASSUMPTION A4: The matrix Ω' is finite and nonsingular. $f^k(0) \neq 0$ for $k \in \{m, f, a, b\}$.

Finiteness of Ω' requires that resource shares are never zero. The matrix is nonsingular provided resource shares are not equal across household sizes. An example of a potential violation would be if parents in households with one fostered child have the exact same resource shares as parents in households with two fostered children, which is unlikely.

The condition that $f^k(0) \neq 0$ requires that the Engel curves for the private assignable and partially assignable goods are continuous and bounded away from zero.

DEFINITION: A *composite* household is a household that contains at least one foster and one non-foster child, or more concisely ($\sigma_a > 0$ and $\sigma_b > 0$).

DEFINITION: A *one-child-type* household is a household that has children, but is not a composite household, or more concisely ($\sigma_a > 0$ and $\sigma_b = 0$) or ($\sigma_a = 0$ and $\sigma_b > 0$) .

ASSUMPTION A5: Assume households with either only foster children, or only non-foster children are observed. With four different person types, there must be at least four different one-child-type households in the data.

For Assumption A5 to hold in this context, it is necessary to observe both one-child-type households with one or two foster children (s_{10} and s_{20}), and also one-child-type households with one or two non-foster children (s_{01} and s_{02}). This requirement is easily met but may be more difficult in other contexts. For example, if one was interested in analysing intrahousehold inequality between widows and non-widow adult women, it is rare to have multiple widows in the same household. In this case, identification could be achieved by observing a one-child-type household with only a widow present, and three different household types with only non-widowed adult women present.

Using one-child-type and composite households in some sense mirrors the central identification assumption of Browning et al. (2013). They use households with single men or single women (one-person-type households) to identify preferences in households with married couples (composite households). Similarly, I use the one-child-type households to impose structure on the composite households. I would however argue that my use of one-child-type households is much weaker than their use of single person households as married men and women likely have different preferences than single men and women, while it is not obvious why foster and non-foster child preferences should differ significantly across one-child-type and composite households.

ASSUMPTION A6: Preferences for clothing for foster and non-foster children are not identical. That is, $f^a(0) \neq f^b(0)$.

Resource shares will be identified by determining whether preferences for children's clothing in the composite households look more like the foster only households, or the non-foster only households. If those preferences are identical, then this method will not work.

Theorem 1. *Let Assumptions A1, A2, A3, A4, A5 and A6 hold for all household sizes s_{ab} in some set S, with one-child-type households $s_{ab} \epsilon \{s_{01}, s_{10}, s_{02}, s_{20}\}$, and composite households s_{ab}.*[11] *Assume the household's Engel curves for the private, assignable good $H^t_{s_{ab}}(y)$ for $t \epsilon \{m, f\}$ and $s_{ab} \epsilon S$ are identified. Assume the household's Engel curve for the private, partially assignable good $H^c_{s_{ab}}$ for $s_{ab} \epsilon S$ is identified. Then resource shares $\eta^t_{s_{ab}}$ for all household members $t \epsilon \{m, f, a, b\}$ in household sizes $s_{ab} \epsilon S$ are identified.*

The above theorem is a generalization of the Dunbar et al. (2013) identification strategy using the SAT restriction. I next show how resource shares can be recovered using the SAP restriction. This theorem is an extension of Theorem 1 in Dunbar et al. (2013).

Define p_m and p_f to be the prices of the private assignable goods. Define p_c to be the price of the private partially assignable goods. The price of all other goods is given by $\tilde{\mathbf{p}}$. As in DLP, define the square matrix $\tilde{\mathbf{A}}_{s_{ab}}$ such that the set of prices given by $A'_{s_{ab}}$ includes the private and partially assignable good prices, p_m, p_f, and p_c, as well as all other prices, given by $\mathbf{A}'_{\mathbf{s_{ab}}}$.

ASSUMPTION B3: Assume each person $t \epsilon \{m, f, a, b\}$ faces the budget constraint defined by $(y, \mathbf{p})$ and has preferences over the private assignable and partially assignable goods, $k \epsilon \{m, f, c\}$ given by the following indirect utility function:

$$V_t(\mathbf{p}, y) = \psi_t\Big[\, \nu(\frac{y}{G^t(\mathbf{p})}) + F^t(\mathbf{p}), \tilde{\mathbf{p}}\Big] \tag{A.26}$$

for some some functions ψ_t, F, and G^t where G^t is nonzero, differentiable, and homogenous of degree one, ν is differentiable and strictly monotonically increasing, $F^t(\mathbf{p})$ is differentiable, homogenous of degree zero, and is such that $\partial F^t(\mathbf{p})/\partial p_t = \phi(\mathbf{p}) \neq 0$. Lastly, ψ_t is differentiable and strictly monotonically increasing in its arguments, and differentiable and homogenous of degree zero in the remaining arguments.

ASSUMPTION B4: For foster and non-foster children, the person-specific expenditure deflators are equal. That is, $G^a = G^b = G^c$, where G^c denotes the expenditure deflator for children.

[11] Resource shares are identified for any composite household provided there is a sufficient number of one-child-type households. In the empirical application, there are ten such households.

By Roy's identity the demand functions for private assignable goods are as follows:

$$\begin{aligned}
h^k(y,\mathbf{p}) =& \frac{v'(\frac{y}{G^k(\mathbf{p})})\frac{y}{G^{k2}(\mathbf{p})}\frac{\partial G^k(\mathbf{p})}{\partial p_k} + \frac{\partial F^k(\mathbf{p})}{\partial p_k}}{v'(\frac{y}{G^k(\mathbf{p})})\frac{1}{G^k(\mathbf{p})}} \\
=& \frac{y}{G^k(\mathbf{p})}\frac{\partial G^k(\mathbf{p})}{\partial p_k} + \frac{\phi(\mathbf{p})}{v'(\frac{y}{G^k(\mathbf{p})})}\frac{y}{y/G^k(\mathbf{p})} = \delta^k(\mathbf{p})y + g\Big(\frac{y}{G^k(\mathbf{p})},\mathbf{p}\Big)y
\end{aligned}$$

Adding the demand functions for foster and non-foster child assignable goods results in the following equation:

$$h^a(y,\mathbf{p}) + h^b(y,\mathbf{p}) = \Big(\delta^a(\mathbf{p}) + \delta^b(\mathbf{p})\Big)y + g\Big(\frac{y}{G^c(\mathbf{p})},\mathbf{p}\Big)y$$

For the private assignable goods for adults, I derive the following household-level demand function.

$$H^k(y,\mathbf{p}) = \delta^k(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p})\eta^k_{s_{ab}}(\mathbf{p})y + g\Big(\frac{\eta^k_{s_{ab}}(\mathbf{p})y}{G^k(A'_{s_{ab}}\mathbf{p})},\mathbf{p}\Big)\eta^k_{s_{ab}}(\mathbf{p})y$$

Let $\eta^c = \sigma_a\eta^a_{s_{ab}} + \sigma_b\eta^b_{s_{ab}}$. Then the household-level demand functions for children's clothing is given by:

$$H^c(y,\mathbf{p}) = \Big(\delta^a(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p}) + \delta^b(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p})\Big)\eta^c_{s_{ab}}(\mathbf{p})y + g\Big(\frac{\eta^c_{s_{ab}}(\mathbf{p})y/(\sigma_a+\sigma_b)}{G^c(\mathbf{A}'_{\mathbf{s_{ab}}}\mathbf{p})}\Big)\eta^c_{s_{ab}}(\mathbf{p})y$$

The Engel curves for adults ($k \in \{m,f\}$) and children are then as follows:

$$H^k_{s_{ab}}(y) = \delta^k_{s_{ab}}\eta^k_{s_{ab}}y + g_{s_{ab}}\Big(\frac{\eta^k_{s_{ab}}y}{G^k_{s_{ab}}}\Big)\eta^k_{s_{ab}}y \tag{A.27}$$

and

$$H^c_{s_{ab}}(y) = \Big(\delta^a_{s_{ab}} + \delta^b_{s_{ab}}\Big)\eta^c_{s_{ab}}y + g\Big(\frac{\eta^c_{s_{ab}}y/(\sigma_a+\sigma_b)}{G^c_{s_{ab}}}\Big)\eta^c_{s_{ab}}y \tag{A.28}$$

ASSUMPTION B5:[12] The function $g_{s_{ab}}$ is twice differentiable. Let $g'_{s_{ab}}(y)$ and $g''_{s_{ab}}(y)$ be the first and second derivatives of $g_{s_{ab}}$. Assume either that $\lambda_{s_{ab}} = \lim_{y\to 0}[y^\zeta g''_{s_{ab}}(y)/g'_{s_{ab}}]^{\frac{1}{1-\zeta}}$ is finite and nonzero for some constant $\zeta \neq 1$ or that $g_{s_{ab}}$ is a polynomial in $\ln y$.

[12] This is Assumption A4 from DLP.

Assumption B5 requires that there be some nonlinearity in the demand function so that g'' is not zero.

ASSUMPTION B6: The ratio of foster and non-foster child resource shares in households with σ_a and $\sigma_{a'}$, and σ_b and $\sigma_{b'}$ foster and non-foster children is constant across household sizes.

$$\frac{\eta^a_{s_{a0}}}{\eta^a_{s_{a+1,0}}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{s_{a+1,b}}} \text{ and } \frac{\eta^b_{s_{0b}}}{\eta^b_{s_{0,b+1}}} = \frac{\eta^b_{s_{ab}}}{\eta^b_{s_{a,b+1}}} \tag{A.29}$$

for σ_a and $\sigma_b \in \{1,2\}$.

This assumption restricts the way in which resource shares vary across household types. In effect, it imposes that resource shares for foster and non-foster children in one-child-type and composite households behave in a similar fashion. Stated differently, this is an independence assumption: the ratio of foster child resource shares in a households with σ_a and σ_{a+1} foster children is independent of the number of *non-foster* children present in those households, and vice versa.

Other studies using the Dunbar et al. (2013) identification strategy have imposed similar restrictions to improve precision in the estimation, but not for identification reasons. For example, Calvi (Forthcoming) parametrizes resource shares in such a way that per person resource shares decrease linearly in the number of household members. In the notation of this study, that would mean assuming $\eta^a_{s_{a,0}} - \eta^a_{s_{a+1,0}} = \eta^a_{s_{ab}} - \eta^a_{s_{a+1,b}}$. On the contrary, I impose that the percent decline is constant, as opposed to the absolute decline. In several specifications, Dunbar et al. (2013) make a similar restriction that per child resource shares decrease linearly in the number of children.

ASSUMPTION B7: The degree of unequal treatment *within* a household with one of each child type is proportional to the degree of unequal treatment *across* households with one foster child or one non-foster child.

$$\frac{\eta^a_{s_{10}}}{\eta^b_{s_{01}}} = \frac{\eta^a_{s_{11}}}{\eta^b_{s_{11}}} \tag{A.30}$$

Similar to Assumption B6, this restriction assumes households with only foster on non-foster children are similar to households with both types of children.

Define the matrix Ω'' by

$$
\Omega'''' = \begin{bmatrix}
\frac{\eta^m_{11}}{\eta^a_{11}} + \frac{\eta^f_{11}}{\eta^a_{11}} + 1 & \frac{\eta^m_{11}}{\eta^b_{11}} + \frac{\eta^f_{11}}{\eta^b_{11}} + 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(\frac{\eta^m_{21}}{\eta^a_{21}} + \frac{\eta^f_{21}}{\eta^a_{21}} + 1) & \frac{\eta^m_{21}}{\eta^b_{21}} + \frac{\eta^f_{21}}{\eta^b_{21}} + 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\eta^m_{12}}{\eta^a_{12}} + \frac{\eta^f_{12}}{\eta^a_{12}} + 1 & 2(\frac{\eta^m_{12}}{\eta^b_{12}} + \frac{\eta^f_{12}}{\eta^b_{12}} + 1) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\eta^m_{22}}{\eta^a_{22}} + \frac{\eta^f_{22}}{\eta^a_{22}} + 1 & \frac{\eta^m_{22}}{\eta^b_{22}} + \frac{\eta^f_{22}}{\eta^b_{22}} + 1 \\
-1 & 0 & \frac{\eta^a_{10}}{\eta^a_{20}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & \frac{\eta^a_{10}}{\eta^a_{20}} & 0 \\
0 & -1 & 0 & 0 & 0 & \frac{\eta^b_{01}}{\eta^b_{02}} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{\eta^b_{01}}{\eta^b_{02}} \\
\frac{1}{\eta^a_{10}} & \frac{-1}{\eta^b_{01}} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

ASSUMPTION B8: The matrix Ω'' is finite and nonsingular.

This is true as long as resource shares are nonzero.

Theorem 2. *Let Assumptions A1, A2, A5, B3, B4, B5, B6, B7, and B8 hold for all household sizes s_{ab} in some set S, with $s_{ab} \in \{s_{01}, s_{10}, s_{02}, s_{20}, s_{11}, s_{12}, s_{21}, s_{22}\}$. Assume the household's Engel curves for the private, assignable good $H^k_{s_{ab}}(y)$ for $k \in \{m, f\}$ for $s_{ab} \in S$ are identified. Assume the household's Engel curve for the private, partially assignable good $H^c_{s_{ab}}$ for $s_{ab} \in S$ is identified. Then resource shares $\eta^t_{s_{ab}}$ for all household members $t \in \{m, f, a, b\}$ in household sizes $s_{ab} \in S$ are identified.*

A.12 Identification Proofs

Proof of Theorem 1

This proof follows the proof of Theorem 2 in Dunbar et al. (2013), and extends it to identify resource shares in the absence of assignable goods for each person type. The proof proceeds in two steps. In the first step, I demonstrate resource shares are identified in the one-child-type households; this follows directly from Dunbar et al. (2013). In the second step, I extend Dunbar et al. (2013) to demonstrate how resource shares are identified in the absence of private assignable goods.

By Assumption A3, the Engel curve functions for the assignable and partially assignable goods are given by Equations (A.24) and (A.25). Let $s_{ab} \in \{s_{10}, s_{20}, s_{01}, s_{02}\}$ be the different one-child-type households. Then since the functions H^k and H^c are identified for $k \in \{m, f\}$, ζ^k_{20}, ζ^k_{02}, and ζ^k_{01} defined as $\zeta^k_{20} = \lim_{y \to 0} H^k_{10}(y)/H^k_{20}(y)$, $\zeta^k_{02} = \lim_{y \to 0} H^k_{10}(y)/H^k_{02}(y)$, and $\zeta^k_{01} = \lim_{y \to 0} H^k_{10}(y)/H^k_{01}(y)$ are all identified. Moreover, $\zeta^a_{20} = \lim_{y \to 0} H^a_{10}(y)/H^a_{20}(y)$ and

$\zeta^b_{02} = \lim_{y\to 0} H^b_{01}(y)/H^b_{02}(y)$ can be identified for foster and non-foster children, respectively.

Then for $k \in \{m, f\}$:

$$\zeta^k_{20} = \frac{f^k(0)\eta^k_{10}}{f^k(0)\eta^k_{20}} = \frac{\eta^k_{10}}{\eta^k_{20}} \quad \text{and} \quad \zeta^k_{02} = \frac{f^k(0)\eta^k_{01}}{f^k(0)\eta^k_{02}} = \frac{\eta^k_{01}}{\eta^k_{02}} \quad \text{and} \quad \zeta^k_{01} = \frac{f^k(0)\eta^k_{10}}{f^k(0)\eta^k_{01}} = \frac{\eta^k_{10}}{\eta^k_{01}}$$

The same ratio for foster and non-foster children in households with only one child type can be identified:

$$\zeta^a_{20} = \frac{(f^a(0)\eta^a_{10} + 0 \times f^b(0)\eta^b_{10})}{(2f^a(0)\eta^a_{20} + 0 \times f^b(0)\eta^b_{20})} = \frac{\eta^a_{10}}{2\eta^a_{20}} \quad \text{and} \quad \zeta^b_{02} = \frac{(0 \times f^a(0)\eta^a_{01} + f^b(0)\eta^b_{01})}{(0 \times f^a(0)\eta^a_{02} + 2f^b(0)\eta^b_{02})} = \frac{\eta^b_{01}}{2\eta^b_{02}}$$

Using that resource shares must sum to one, the following equations can be written, first for households with only non-foster children:

$$\begin{aligned}
&\zeta^m_{s_{20}}\eta^m_{s_{20}} + \zeta^f_{s_{20}}\eta^f_{s_{20}} + \zeta^a_{s_{20}}\sigma_a\eta^a_{s_{20}} = \eta^m_{10} + \eta^f_{10} + \eta^a_{10} = 1 \\
&\zeta^m_{s_{20}}\eta^m_{s_{20}} + \zeta^f_{s_{20}}\eta^f_{s_{20}} + \zeta^a_{s_{20}}(1 - \eta^m_{s_{20}} - \eta^f_{s_{20}}) = 1 \\
&(\zeta^m_{s_{20}} - \zeta^a_{s_{20}})\eta^m_{s_{20}} + (\zeta^f_{s_{20}} - \zeta^a_{s_{20}})\eta^f_{s_{20}} = 1 - \zeta^a_{s_{20}}
\end{aligned}$$

and then for households with only foster children:

$$\begin{aligned}
&\zeta^m_{s_{02}}\eta^m_{s_{02}} + \zeta^f_{s_{02}}\eta^f_{s_{02}} + \zeta^b_{s_{02}}\sigma_b\eta^b_{s_{02}} = \eta^m_{01} + \eta^f_{01} + \eta^b_{01} = 1 \\
&\zeta^m_{s_{02}}\eta^m_{s_{02}} + \zeta^f_{s_{02}}\eta^f_{s_{02}} + \zeta^b_{s_{02}}(1 - \eta^m_{s_{02}} - \eta^f_{s_{02}}) = 1 \\
&(\zeta^m_{s_{02}} - \zeta^b_{s_{02}})\eta^m_{s_{02}} + (\zeta^f_{s_{02}} - \zeta^b_{s_{02}})\eta^f_{s_{02}} = 1 - \zeta^b_{s_{02}}
\end{aligned}$$

These above equations for $t \in \{m, f\}$, give the matrix equation

$$\begin{bmatrix}
\zeta^m_{20} & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \zeta^f_{20} & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \zeta^m_{02} & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \zeta^f_{02} & -1 \\
0 & -1 & 0 & 0 & 0 & \zeta^m_{01} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \zeta^f_{01} \\
\zeta^m_{20} - \zeta^a_{20} & 0 & \zeta^f_{20} - \zeta^a_{20} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \zeta^m_{02} - \zeta^b_{02} & 0 & \zeta^f_{02} - \zeta^b_{02} & 0
\end{bmatrix} \times \begin{bmatrix} \eta^m_{20} \\ \eta^m_{10} \\ \eta^f_{20} \\ \eta^f_{10} \\ \eta^m_{02} \\ \eta^m_{01} \\ \eta^f_{02} \\ \eta^f_{01} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 - \zeta^a_{20} \\ 1 - \zeta^b_{02} \end{bmatrix}$$

The 8×8 matrix in this equation equals the previously defined matrix Ω' which was assumed to be nonsingular. Therefore the system can be solved for $\eta^m_{s_{a0}}$, $\eta^m_{s_{0b}}$, $\eta^f_{s_{a0}}$, and $\eta^f_{s_{0b}}$. Non-foster child resource shares and foster child resource shares can then be identified for one-child-type only households by $\eta^a_{s_{a0}} = (1 - \eta^m_{s_{a0}} - \eta^f_{s_{a0}})/\sigma_a$ and $\eta^b_{s_{0b}} = (1 - \eta^m_{s_{0b}} - \eta^f_{s_{0b}})/\sigma_b$.

I now show resource shares are identified in any given composite household. Recall that the functions H^k are identified for $k \in \{m, f\}$. It follows that for any household type s_{ab}, $\zeta^k_{s_{ab}}$

defined as $\zeta^k_{s_{ab}} = \lim_{y\to 0} H^k_{10}(y)/H^k_{s_{ab}}(y)$ can be identified.

Then for $k \epsilon \{m, f\}$:

$$\zeta^k_{s_{ab}} = \frac{f^k(0)\eta^k_{10}}{f^k(0)\eta^k_{s_{ab}}} = \frac{\eta^k_{10}}{\eta^k_{s_{ab}}}$$

With η^k_{10} already identified, resource shares for men and women in the composite household types can be recovered. This is a simple extension of Dunbar et al. (2013) where there are more household types than individual types.

I now aim to separately identify non-foster and foster child resource shares in households with both types of children. Define $\zeta^a_{s_{ab}}$ as follows: $\zeta^a_{s_{ab}} = \lim_{y\to 0} H^c_{s_{ab}}(y)/H^c_{10}(y)$. Moreover, define $\zeta^b_{01} = \lim_{y\to 0} H^c_{01}(y)/H^c_{10}(y)$. Then we can write:

$$\zeta^a_{s_{ab}} = \frac{f^a(0)\eta^a_{s_{ab}} + f^b(0)\eta^b_{s_{ab}}}{f^a(0)\eta^a_{10}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{10}} + \frac{f^b(0)\eta^b_{s_{ab}}}{f^a(0)\eta^a_{10}} \tag{A.31}$$

Furthermore,

$$\zeta^b_{01} = \frac{f^b(0)\eta^b_{01}}{f^a(0)\eta^a_{10}} \to \frac{f^b(0)}{f^a(0)} = \frac{\zeta^b_{01}\eta^a_{10}}{\eta^b_{01}} = \kappa$$

where η^a_{10} and η^b_{01} have already been identified. Thus, the ratio $f^b(0)/f^a(0) = \kappa$ is identified. Substituting κ into equation (A.31) results in the following expression:

$$\zeta^a_{s_{ab}} = \frac{\eta^a_{s_{ab}}}{\eta^a_{10}} + \kappa\frac{\eta^b_{s_{ab}}}{\eta^a_{10}} \tag{A.32}$$

where only $\eta^a_{s_{ab}}$ and $\eta^b_{s_{ab}}$ are unknown. Then since resource shares for men and women have already been identified for households of type s_{ab}, and because resource shares sum to one, we can solve for $\eta^a_{s_{ab}}$ and $\eta^b_{s_{ab}}$. This has a unique solution following Assumption A6.

Proof of Theorem 2

The proof proceeds in two steps. In the first step, I demonstrate resource shares are identified in the one-child-type households; this follows directly from Dunbar et al. (2013). In the second step, I extend Dunbar et al. (2013) to demonstrate how resource shares can be identified in the absence of private assignable goods.

By Assumption B3, Engel curves for the private assignable goods for men and women are given by Equation (A.27) and by Assumptions B3 and B4, the Engel curve for the private partially assignable good is given by Equation (A.28). Define $\tilde{h}^k_{s_{ab}}(y) = \partial[H^k_{s_{ab}}(y)/y]\partial y$ and $\lambda_{s_{ab}} = \lim_{y\to 0}[y^\zeta g^{''}_{s_{ab}}(y)/g^{'}_{s_{ab}}]^{\frac{1}{1-\zeta}}$, where $\zeta \neq 1$ (the log polynomial case, where $\zeta = 1$ is considered in the second case).

Case 1: $g_{s_{ab}}$ is not a polynomial in logarithms.

Let $\sigma_c = \sigma_a + \sigma_b$ be the total number of children. Then since $H^k_{s_{ab}}(y)$ are identified for $k \epsilon \{m, f, c\}$, we can identify $\kappa^k_{s_{ab}}$ for men and women defined as follows:

$$\begin{aligned}
\kappa^k_{s_{ab}} =& \left(y^\zeta \frac{\partial \tilde{h}^k_{s_{ab}}(y)/\partial y}{\tilde{h}^k_{s_{ab}}(y)}\right)^{\frac{1}{1-\zeta}} \\
=& \left(\Big(\frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}}\Big)^{-\zeta}\Big(\frac{\eta^k_{s_{ab}} y}{G^k_{s_{ab}}}\Big)^\zeta \Big[g^{''}_{s_{ab}}\Big(\frac{\eta^k_{s_{ab}} y}{G^k_{s_{ab}}}\Big)\frac{\eta^{k^3}_{s_{ab}}}{G^{k^2}_{s_{ab}}}\Big] / \Big[g^{'}_{s_{ab}}\Big(\frac{\eta^k_{s_{ab}} y}{G^k_{s_{ab}}}\Big)\frac{\eta^{k^2}_{s_{ab}}}{G^k_{s_{ab}}}\Big]\right)^{\frac{1}{1-\zeta}} \\
=& \frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}}\left(y^\zeta_{k,s_{ab}} \frac{g^{''}_{s_{ab}}(y_{k,s_{ab}})}{g^{'}_{s_{ab}}(y_{k,s_{ab}})}\right)^{\frac{1}{1-\zeta}}
\end{aligned}$$

and for children:

$$\begin{aligned}
\kappa^c_{s_{ab}} =& \left(y^\zeta \frac{\partial \tilde{h}^c_{s_{ab}}(y)/\partial y}{\tilde{h}^c_{s_{ab}}(y)}\right)^{\frac{1}{1-\zeta}} = \\
& \left(\Big(\frac{\eta^c_{s_{ab}}}{G^c_{s_{ab}} s_c}\Big)^{-\zeta}\Big(\frac{\eta^c_{s_{ab}} y}{G^c_{s_{ab}} s_c}\Big)^\zeta \Big[g^{''}_{s_{ab}}\Big(\frac{\eta^c_{s_{ab}} y}{G^c_{s_{ab}} s_c}\Big)\frac{\eta^{c^3}_{s_{ab}}}{G^{c^2}_{s_{ab}} s_c^2}\Big] / \Big[g^{'}_{s_{ab}}\Big(\frac{\eta^c_{s_{ab}} y}{G^c_{s_{ab}} s_c}\Big)\frac{\eta^{c^2}_{s_{ab}}}{G^c_{s_{ab}} s_c}\Big]\right)^{\frac{1}{1-\zeta}} \\
=& \frac{\eta^c_{s_{ab}}}{G^c_{s_{ab}} s_c}\left(y^\zeta_{c,s_{ab}} \frac{g^{''}_{s_{ab}}(y_{c,s_{ab}})}{g^{'}_{s_{ab}}(y_{c,s_{ab}})}\right)^{\frac{1}{1-\zeta}}
\end{aligned}$$

Then for $k \epsilon \{m, f\}$, $\kappa^k_{s_{ab}}(0) = \frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}}\lambda_{s_{ab}}$, and we can identify $\rho^k_{s_{ab}}(y)$ defined as:

$$\rho^k_{s_{ab}}(y) = \frac{\tilde{h}^k_{s_{ab}}(y/\kappa^k_{s_{ab}}(0))}{\kappa^k_{s_{ab}}(0)} = g^{'}_{s_{ab}}\Big(\frac{y}{\lambda_{s_{ab}}}\Big)\frac{\eta^k_{s_{ab}}}{\lambda_{s_{ab}}}$$

and for $k = c$, $\kappa^c_{s_{ab}}(0) = \frac{\eta^c_{s_{ab}}}{G^c_{s_{ab}} s_c}\lambda_{s_{ab}}$, and we can identify $\rho^c_{s_{ab}}(y)$ defined as:

$$\rho^c_{s_{ab}}(y) = \frac{\tilde{h}^c_{s_{ab}}\left(y/\kappa^c_{s_{ab}}(0)\right)}{\kappa^c_{s_{ab}}(0)} = g^{'}_{s_{ab}}\Big(\frac{y}{\lambda_{s_{ab}}}\Big)\frac{\eta^c_{s_{ab}}}{\lambda_{s_{ab}}}$$

and we can write $\gamma^k_{s_{ab}}$ for $k \epsilon \{m,f\}$:

$$\gamma^k_{s_{ab}} = \frac{\tilde{\rho}^k_{s_{ab}}}{\tilde{\rho}^c_{s_{ab}}} = \left(g^{'}_{s_{ab}}\left(\frac{y}{\lambda_{s_{ab}}}\right)\frac{\eta^k_{s_{ab}}}{\lambda_{s_{ab}}}\right) \Big/ \left(g^{'}_{s_{ab}}\left(\frac{y}{\lambda_{s_{ab}}}\right)\frac{\eta^c_{s_{ab}}}{\lambda_{s_{ab}}}\right) = \frac{\eta^k_{s_{ab}}}{(\sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}})} \tag{A.33}$$

Case 2: Before proceeding with the proof, I examine the case where $g_{s_{ab}}$ is a polynomial in logarithms (the end result will be Equation (A.33) and I will proceed with both cases simultaneously afterwards). Suppose $g_{s_{ab}}$ is a polynomial of degree λ in logarithms. Then

$$g_{s_{ab}}\left(\frac{\eta^k_{s_{ab}} y}{G^k}\right) = \sum_{l=0}^{\lambda}\left(\ln\left(\frac{\eta^k_{s_{ab}}}{G^k_{s_{ab}}}\right) + \ln(y)\right)^l c_{s_{ab},l}$$

Then for $k \epsilon \{m,f\}$:

$$\begin{aligned}\gamma^k_{s_{ab}} =& \left(\frac{\partial^\lambda [H^k_{s_{ab}}(y)/y]}{\partial(\ln y)^\lambda}\right) \Big/ \left(\frac{\partial^\lambda [H^c_{s_{ab}}(y)/y]}{\partial(\ln y)^\lambda}\right) = \\ & \frac{c_{s_{ab},\lambda}\eta^k_{s_{ab}}}{c_{s_{ab},\lambda}(\sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}})} = \frac{\eta^k_{s_{ab}}}{(\sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}})}\end{aligned} \tag{A.34}$$

which is the same as Equation (A.33). Then since resource shares must sum to one:

$$\begin{aligned}&\gamma^m_{s_{ab}}(\sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}}) + \gamma^f_{s_{ab}}(\sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}}) + \sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}} = \\ &\eta^m_{s_{ab}} + \eta^f_{s_{ab}} + \sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}} = 1 \\ &\sigma_a \eta^a_{s_{ab}}(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1) + \sigma_b \eta^b_{s_{ab}}(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1) = 1\end{aligned} \tag{A.35}$$

For one-child-type households, σ_a or σ_b equals zero, and Equation (A.35) simplifies significantly. For households that only have foster children, Equation (A.35) can be written as follows:

$$\sigma_a \eta^a_{s_{ab}}(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1) = 1$$

which can be solved for $\eta^a_{s_{ab}} = \frac{1}{\sigma_a(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1)}$. Similarly, $\eta^b_{s_{ab}} = \frac{1}{\sigma_b(\gamma^m_{s_{ab}} + \gamma^f_{s_{ab}} + 1)}$.

With resource shares for foster and non-foster children identified, resource shares for men and women in the one-child-type households can then be solved for since $\eta^t_{s_{ab}} = \gamma^t_{s_{ab}}(\sigma_a \eta^a_{s_{ab}} + \sigma_b \eta^b_{s_{ab}})$ for $t \epsilon \{m,f\}$.

I next move to the composite households $s_{ab} \epsilon \{s_{11}, s_{21}, s_{12}, s_{22}\}$. Note that now, for each household type, resource shares for both foster and non-foster children need to be identified (η^a and η^b). For the one-child-type households, one of those two parameters was zero. From Equation (A.35) I can write the following four equations:

$$
\begin{bmatrix}
1+\gamma_{11}^m+\gamma_{11}^f & 1+\gamma_{11}^m+\gamma_{11}^f & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(1+\gamma_{21}^m+\gamma_{21}^f) & 1+\gamma_{21}^m+\gamma_{21}^f & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1+\gamma_{12}^m+\gamma_{12}^f & 2(1+\gamma_{12}^m+\gamma_{12}^f) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1+\gamma_{22}^m+\gamma_{22}^f & 1+\gamma_{22}^m+\gamma_{22}^f
\end{bmatrix}
\times
\begin{bmatrix} \eta_{11}^a \\ \eta_{11}^b \\ \eta_{21}^a \\ \eta_{21}^b \\ \eta_{12}^a \\ \eta_{12}^b \\ \eta_{22}^a \\ \eta_{22}^b \end{bmatrix}
=
\begin{bmatrix} 1 \\ 1 \\ 1 \\ \frac{1}{2} \end{bmatrix}
$$

Clearly the above system is under-identified as there are eight unknowns and only four equations. I now impose Assumptions B6 and B7, which add an additional five equations to the system. Note that the resource shares for the one-child-type households have already been identified (i.e. η_{10}^a is known at this point). This results in the following system of nine equations

$$
\begin{bmatrix}
1+\gamma_{11}^m+\gamma_{11}^f & 1+\gamma_{11}^m+\gamma_{11}^f & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(1+\gamma_{21}^m+\gamma_{21}^f) & 1+\gamma_{21}^m+\gamma_{21}^f & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1+\gamma_{12}^m+\gamma_{12}^f & 2(1+\gamma_{12}^m+\gamma_{12}^f) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1+\gamma_{22}^m+\gamma_{22}^f & 1+\gamma_{22}^m+\gamma_{22}^f \\
-1 & 0 & \frac{\eta_{10}^a}{\eta_{20}^a} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & \frac{\eta_{10}^a}{\eta_{20}^a} & 0 \\
0 & -1 & 0 & 0 & 0 & \frac{\eta_{01}^b}{\eta_{02}^b} & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & \frac{\eta_{01}^b}{\eta_{02}^b} \\
\frac{1}{\eta_{10}^a} & \frac{-1}{\eta_{01}^b} & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix} \eta_{11}^a \\ \eta_{11}^b \\ \eta_{21}^a \\ \eta_{21}^b \\ \eta_{12}^a \\ \eta_{12}^b \\ \eta_{22}^a \\ \eta_{22}^b \end{bmatrix}
=
\begin{bmatrix} 1 \\ 1 \\ 1 \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

This eight by nine matrix is equal to the matrix Ω'' defined earlier with $\gamma_{s_{ab}}^t = \frac{\eta_{s_{ab}}^m}{\sigma_a \eta_{s_{ab}}^a + \sigma_b \eta_{s_{ab}}^b}$, which is nonsingular by Assumption B8. The system can therefore be solved for $\eta_{s_{ab}}^a$ and $\eta_{s_{ab}}^b$. Resource shares for men and women can then be solved for since $\eta_{s_{ab}}^t = \gamma_{s_{ab}}^t(\sigma_a \eta_{s_{ab}}^a + \sigma_b \eta_{s_{ab}}^b)$ for $t \in \{m, f\}$.

References

Ainsworth, Martha, "Economic Aspects of Child Fostering in Cote d'Ivoire," *Research in Population Economics*, 1995, *8*, 25–62. [18]

— **and Deon Filmer**, "Inequalities in Children's Schooling: AIDS, Orphanhood, Poverty, and Gender," *World Development*, 2006, *34* (6), 1099–1128. [2]

Akresh, Richard, "Flexibility of Household Structure: Child Fostering Decisions in Burkina Faso," *Journal of Human Resources*, 2009, *44* (4), 976–997. [6], [18]

Bargain, Olivier, Guy Lacroix, and Luca Tiberti, "Validating the Collective Model of Household Consumption Using Direct Evidence on Sharing," 2018. [10]

Beck, Simon, Philippe De Vreyer, Sylvie Lambert, Karine Marazyan, and Abla Safir, "Child fostering in Senegal," *Journal of Comparative Family Studies*, 2015, pp. 57–73. [5]

Bourguignon, François, Martin Browning, and Pierre-André Chiappori, "Efficient Intra-household Allocations and Distribution Factors: Implications and Identification," *The Review of Economic Studies*, 2009, *76* (2), 503–528. [19]

Brown, Caitlin, Rossella Calvi, and Jacob Penglase, "Sharing the Pie: Undernutrition, Intra-Household Allocation, and Poverty," 2018. [20]

Browning, Martin and Pierre-André Chiappori, "Efficient Intra-household Allocations: A General Characterization and Empirical Tests," *Econometrica*, 1998, pp. 1241–1278. [19], [20]

_, Pierre-Andre Chiappori, and Arthur Lewbel, "Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power," *The Review of Economic Studies*, 2013, pp. 1267–1303. [37]

_, Pierre-André Chiappori, and Yoram Weiss, *Economics of the Family*, Cambridge University Press, 2014. [19]

Calvi, Rossella, "Why Are Older Women Missing in India? The Age Profile of Bargaining Power and Poverty," *Journal of Political Economy*, Forthcoming. [40]

Case, Anne, Christina Paxson, and Joseph Ableidinger, "Orphans in Africa: Parental Death, Poverty, and School Enrollment," *Demography*, 2004, *41* (3), 483–508. [2]

Deaton, Angus and John Muellbauer, *Economics and Consumer Behavior*, Cambridge University Press, 1980. [22]

Dunbar, Geoffrey R, Arthur Lewbel, and Krishna Pendakur, "Children's Resources in Collective Households: Identification, Estimation, and an Application to Child Poverty in Malawi," *The American Economic Review*, 2013, *103* (1), 438–471. [19], [21], [33], [34], [35], [38], [40], [41], [43]

Lechene, Valérie, Krishna Pendakur, and Alex Wolf, "OLS Estimation of the Intra-Household Distribution of Consumption," 2019. [10]

Rangel, Marcos and Duncan Thomas, "Decision-Making in Complex Households," Working Paper 26511, National Bureau of Economic Research November 2019. [19]

Serra, Renata, "Child Fostering in Africa: When Labor and Schooling Motives May Coexist," *Journal of Development Economics*, 2009, *88* (1), 157–170. [5], [18]